Accurate Prediction of Dynamic Fracture with Peridynamics

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Fracture Mechanics Theory and Dynamic Fracture

• Onset of crack growth can be accurately predicted
• Crack growth speed and direction cannot!

Wanted: A successful method for simulating Dynamic Fracture

• Such a method must be able to reproduce the characteristic phenomena of Dynamic Fracture.
• All current simulation methods severely fail this test of efficacy
  - The governing PDEs break down at cracks.
Characteristic Phenomena of Dynamic Fracture:

1. The Mirror, Mist, Hackle sequence of textures on the fracture surface;
2. A steady, limiting crack speed;
3. The transition from stable to unstable crack growth;
4. Crack branching;
5. Fragment size distribution;
6. The specific angle of cracking following impact of a notched plate (the Kaltoff-Winkler experiment);
7. The multiple, unstable cracking modes of fiber-reinforced composites;
8. Membrane bursting;
9. Unstable peeling and tearing of thin sheets.
1, 2. Dynamic fracture in PMMA: J. Fineberg & M. Marder, Phys Rpts 313 (1999) 1-108

Mirror-mist-hackle transition

Microbranching

Limiting Crack Speed

Time $t$ (µsec)

Velocity $v$ (m/sec)
3. Transition from Stable to Unstable Crack Growth

- At what load and what length does crack growth change from stable (slow) to unstable (fast)?

Crack just before transition to Unstable (photo courtesy Boeing)
4. Crack Branching

At high levels of stress intensity factor (e.g., at a round notch) a crack propagates at higher stress levels and sequential crack branching can occur.

Crack branching in notched specimens of soda-lime glass, pulled vertically (specimens 3” x 1” x 0.05”; load stress increasing left to right)

5. Fragmentation: Ductile Aluminum Ring*

1100-0 Al ring has initial radial velocity of 200 m/s.
- 32mm diameter, 1mm x 1mm cross-section.
- Rapid acceleration by electromagnetic pulse.
- Recovered 11-13 fragments per shot over range of initial velocities 182 – 220 m/s.

6. Dynamic fracture in a hard steel plate: Kalthoff-Winkler Experiment

Dynamic fracture in maraging steel (Kalthoff & Winkler, 1988)
• Mode-II loading at notch tips results in mode-I cracks at 70° angle.

Experimental Results

70°
7. Splitting and fracture mode changes in Fiber-reinforced Composites

The distribution of fiber directions between plies strongly influences the way cracks grow.

Typical crack growth in a notched laminate (photo courtesy Boeing)
Peeling of adhesive film from rigid substrate exhibits characteristic tearing behavior.

9b. Instability in the slow tearing of an elastic membrane

Peridynamics Governing Equation

- PDEs are replaced by the following integral equation:
  \[ \rho \ddot{u}(x, t) = \int_H f(u' - u, x' - x) dV' - b(x, t) \]  
  \( (1) \)

- Eq. (1) replaces the classical PDE:
  \[ \rho \ddot{u}(x, t) = \nabla \cdot \sigma(x, t) - b(x, t) \]  
  \( (2) \)

- New approach is an extension of classical continuum mechanics:
  - PD, Eq. (1) is not derivable from Eq. (2);
  - PD converges to (2) in the limit \( \delta \to 0 \) (J. Elasticity 2008);
  - PD holds regardless of discontinuities;
  - PD entails force & displacement, not stress & strain;
  - PD includes interaction of material particles over a finite distance

- PD is intrinsically non-local,
- Eq. (1) is a “Continuum version of molecular dynamics”
Numerical method and material model incorporate damage at the “bond” level

- Finite sum replaces integral: method is meshless and Lagrangian.
- Force parameters come from measurable elastic-plastic and fracture data.
- Simulate explicit time integration dynamics or static equilibrium; extension to implicit dynamics is underway.

\[ \rho \ddot{u}_i^n = \sum_{k \in H} f(u_k^n - u_i^n, x_k - x_i) \Delta V_i + b(x_i, t) \]

All material-specific behavior is contained in the force density, \( f \).
Fracture Phenomena *Emerge from a Peridynamics Simulation*

Peridynamics is a history-dependent theory in which crack initiation and growth, and all associated phenomena, *emerge spontaneously*, in an *unguided* fashion, simply from the choice of system geometry, ICs, BCs, and the constitutive model.
1. Dynamic fracture in PMMA: Damage features

- Microbranching*
- Mirror-mist-hackle transition*

Peridynamics damage

Peridynamics crack surfaces

* J. Fineberg & M. Marder, Physics Reports 313 (1999) 1-108
2. Dynamic fracture in PMMA: Crack tip velocity

- Crack velocity increases to a critical value, then oscillates.

Peridynamics

Experiment*

3. Peridynamics model of Ribbed Plate

Load

Defect

Damage control feature

Load

Peridynamic model
3. Simulation of damage control feature and crack instability

Crack trajectory after instability

Crack just before transition to unstable (photo courtesy Boeing)
4. Dynamic Brittle Fracture in Glass

• Soda-lime glass plate
  - Dimensions: 3” x 1” x 0.05”
  - Density: 2.44 g/cm$^3$
  - Elastic Modulus: 79.0 GPa

• Notch at top; apply tension

• Discretization (finest)
  - Mesh spacing: 35 microns
  - Approx. 82 million particles
  - Time: 50 microseconds (20k timesteps)
  - 6 hours on 65k cores using DAWN (LLNL): IBM BG/P System
  - 500 teraflops; 147,456 cores

• Simulation by M. Parks (SNL), F. Bobaru & Y. Ha (Nebraska)
4. Crack Branching in Brittle in Glass

Crack branching *emerges* in the Peridynamics simulation of the experimental specimen and loading conditions:

- Branching is qualitatively identical to experiment;
- Onset of simulated branching is earlier than experiment.

5. Fragmentation of a Ductile Aluminum Ring*

Experiment *:

- Recovered 11-13 fragments per shot over range of initial velocities 182 – 220 m/s.

Peridynamics model produces 12 fragments.

Dynamic fracture in maraging steel (Kalthoff & Winkler, 1988)

- 3D Peridynamics model reproduces the 70° crack angle.

Peridynamics

Vertical displacement contours

Experiment
7. Splitting and fracture mode changes in Fiber-reinforced Composites

- Distribution of fiber directions between plies strongly influences the way cracks grow.

Typical crack growth in a notched laminate bar

(a) Quasi-isotropic;
(b) Additional +/- 45°;
(c) Extra +/- 45°;
(d) Mostly 0° (along length)

Typical crack growth in a notched laminate (photo courtesy Boeing)
8. Dynamic fracture in membranes

Bursting of a balloon by a bullet. Time increases right to left.
High speed photo by H. Edgerton (MIT collection)

Peridynamics model of a balloon penetrated by a fragment.
(Time sequence numbered.)
9a. Peeling and tearing

Peridynamics: Unsupported sheet, pinned along 3 edges (Computers and Structures, 2005)

9a. *Interaction of 2 cracks: Peeling of a sheet*

- Pull upward on part of a free edge – other 3 edges are fixed.

"Experimental data"
9b. Instability in the slow tearing of an elastic membrane


Conclusion

Fracture phenomena spontaneously emerge in peridynamics simulations simply as a result of the choice of system geometry, ICs, BCs, and material model.

No supplemental kinetic relations are required; cracks grow in an unguided fashion.

In this sense peridynamics is predictive of dynamic fracture phenomena.
Peridynamic Can Use Diverse Constitutive Models

- Peridynamics can model large-strain, rate-dependent, strain-hardening plasticity.

Necking in a 6061-Aluminum Bar

Simulation with state-based peridynamic implementation of large-deformation, strain-hardening, rate-dependent material model.

- Force-strain plot shows stable response with decreasing load.
Convergence in fragmentation: Expanding 3D annulus

- Outer radius = 100 mm
- Thickness = 10 mm
- Radial velocity = 600 m/s

- Grid contains 1% random perturbations to act as seed.

For each choice of the horizon, select the material model parameters such that the energy release rate is the same for all.
Convergence in fragmentation:
Same problem with 4 different grid spacings

\[ \Delta x = 3.33 \text{ mm} \]
\[ \Delta x = 1.43 \text{ mm} \]
\[ \Delta x = 2.00 \text{ mm} \]
\[ \Delta x = 1.00 \text{ mm} \]

\[ \delta = 3 \Delta x \]

Horizon = 3x grid spacing

Colors are for visualization
Convergence in fragmentation:
Fragment mass distribution

Cumulative distribution function
for 4 grid spacings

<table>
<thead>
<tr>
<th>Δx (mm)</th>
<th>Mean fragment mass (g)</th>
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<tbody>
<tr>
<td>3.33</td>
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<tr>
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Solution appears essentially converged