Deviatoric Constitutive Model
Domain of Strain Rate Validity

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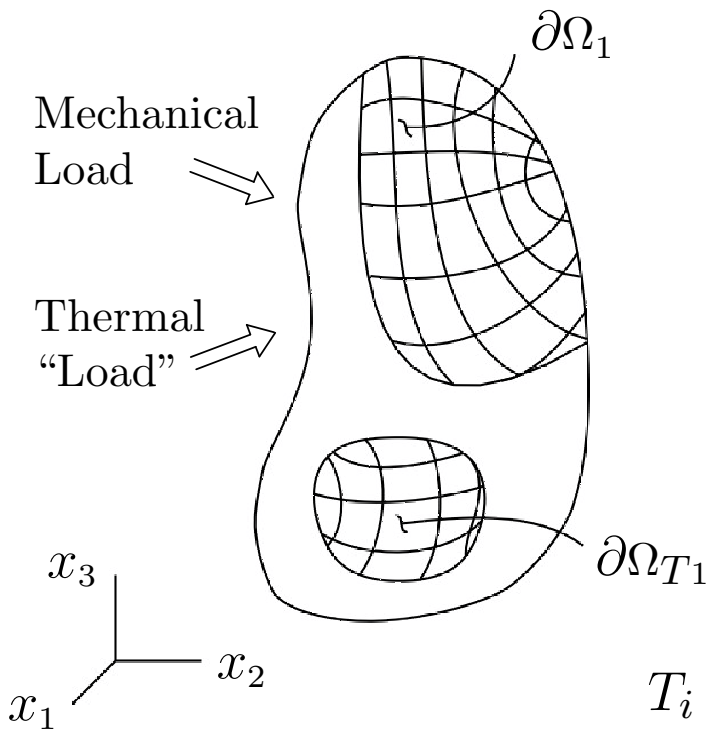
Initial Boundary Value Problem



CM $$\frac{\partial \rho}{\partial t} + (\rho u_i)_{,i} = 0$$

CLM $$\frac{\partial}{\partial t}(\rho u_i) + \left(\rho u_i u_j\right)_{,j} = \sigma_{ji,j} + \rho f_i$$

CE $$\frac{\partial}{\partial t}(\rho h) + (\rho h u_i)_{,i} = \sigma_{ji} D_{ij} - q_{i,i} + \rho r + \frac{dp}{dt} + p u_{i,i}$$

$$D_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right)$$

$$u_i = \widehat{u}_i \ \text{ on } \ \partial\Omega_1 \qquad \theta = \widehat{\theta} \ \text{ on } \ \partial\Omega_{T2}$$

$$T_i = \sigma_{ji} n_j = \widehat{T} \ \text{ on } \ \partial\Omega_2 \qquad k_{ij}\theta_{,i} n_j = \widehat{T}_T \ \text{ on } \ \partial\Omega_{T2}$$

$$\begin{aligned} \theta(x_k, t) &= 0 \\ u_i(x_k, t) &= 0 \qquad \text{for } \ t < 0 \\ \sigma_{ij}(x_k, t) &= 0 \end{aligned}$$

$$\partial\Omega_1 \cup \partial\Omega_2 = \partial\Omega \quad \text{and} \quad \partial\Omega_1 \cap \partial\Omega_2 = \emptyset$$
$$\partial\Omega_{T1} \cup \partial\Omega_{T2} = \partial\Omega \quad \text{and} \quad \partial\Omega_{T1} \cap \partial\Omega_{T2} = \emptyset$$

$$\boxed{\sigma_{ij} = \sigma_{ij}\left\{\epsilon_{kl}, \dot{\epsilon}_{kl}, T, T_k, \ldots\right\}}$$

Classical Plasticity

4 Parts: (1) stress-strain relationship, (2) yield criterion, (3) flow rule, (4) hardening rule

Preston-Tonks-Wallacs (PTW)

$$G = G_0 \left(1 - \alpha \widehat{T}\right) \qquad \dot{\xi} = \frac{1}{2}\left(\frac{4\pi\rho}{3M}\right)^{\frac{1}{3}} \sqrt{\frac{G}{\rho}}$$

$$\widehat{\tau}_s = \max\left\{S_0 - (S_0 - S_\infty)\,\mathrm{erf}\left[\kappa \widehat{T} \ln\left(\frac{\gamma\dot{\xi}}{\dot{\psi}}\right)\right],\ S_0\left(\frac{\dot{\psi}}{\gamma\dot{\xi}}\right)^{\beta}\right\}$$

$$\widehat{\tau}_y = \max\left\{y_0 - (y_0 - y_\infty)\,\mathrm{erf}\left[\kappa \widehat{T} \ln\left(\frac{\gamma\dot{\xi}}{\dot{\psi}}\right)\right],\ \min\left[y_1\left(\frac{\dot{\psi}}{\gamma\dot{\xi}}\right)^{y_2},\ S_0\left(\frac{\dot{\psi}}{\gamma\dot{\xi}}\right)^{\beta}\right]\right\}$$

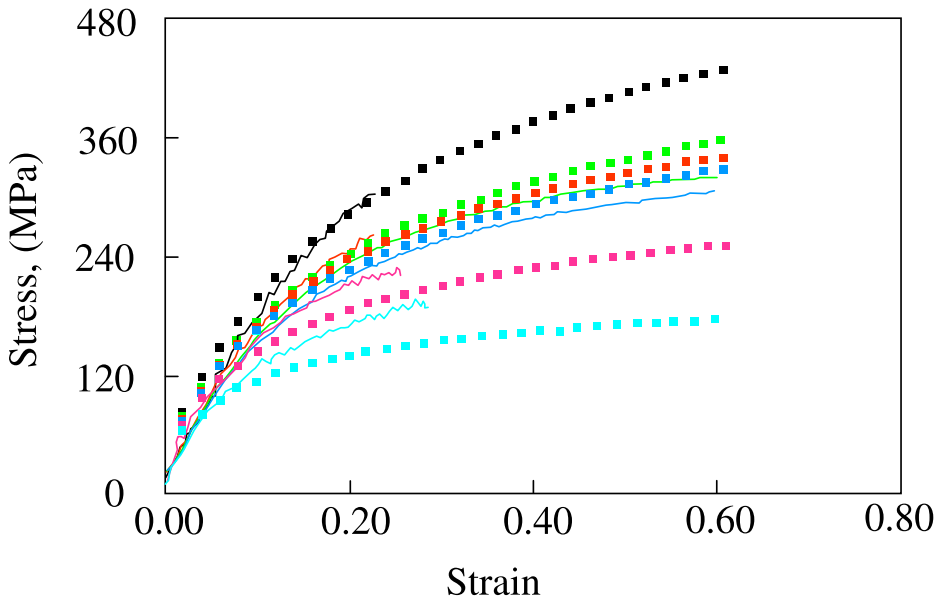
$$\widehat{\tau} = \widehat{\tau}_s + \frac{1}{p}(s_0 - \widehat{\tau}_y) \ *$$

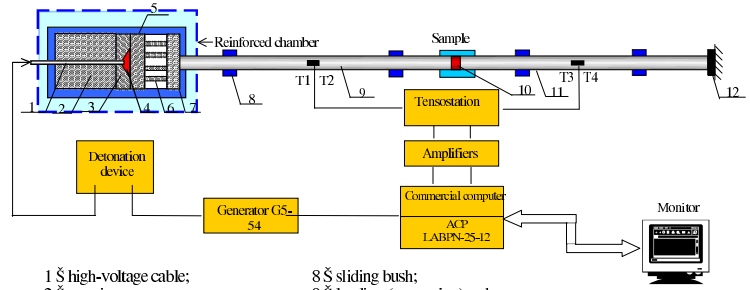
$$\ln\left[1 - \left[1 - \exp\left(-p\frac{\widehat{\tau}_s - \widehat{\tau}_y}{S_0 - \widehat{\tau}_y}\right)\right] \exp\left\{-\frac{p\theta_0\psi}{(S_0 - \widehat{\tau}_y)\left[\exp\left(p\frac{\widehat{\tau}_s - \widehat{\tau}_y}{S_0 - \widehat{\tau}_y}\right) - 1\right]}\right\}\right]$$

$$\sigma = 2\,\widehat{\tau}\,G$$

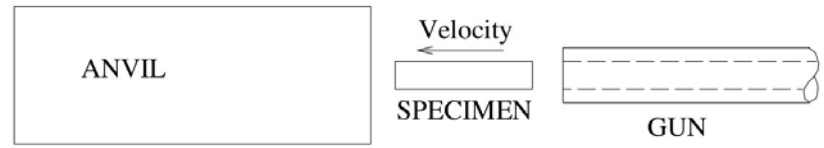
Fifteen input parameters: p, θ_0, κ, γ, α, G_0, T_m, M, S_0, S_∞, y_0, y_∞, β, y_1, y_2

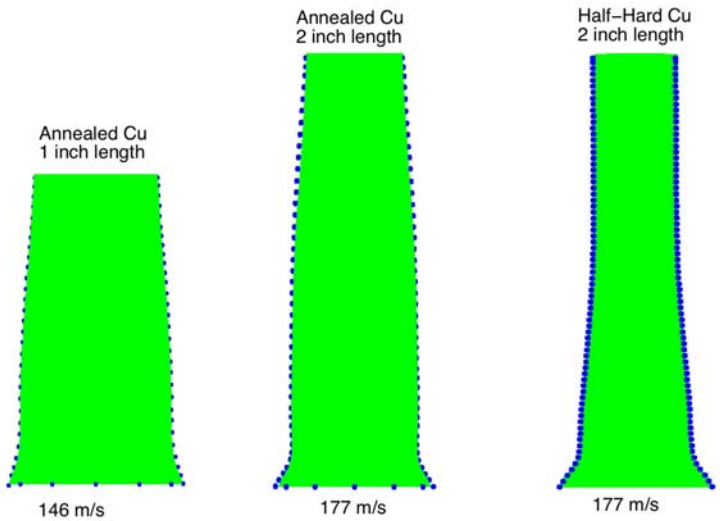
Model Parameters

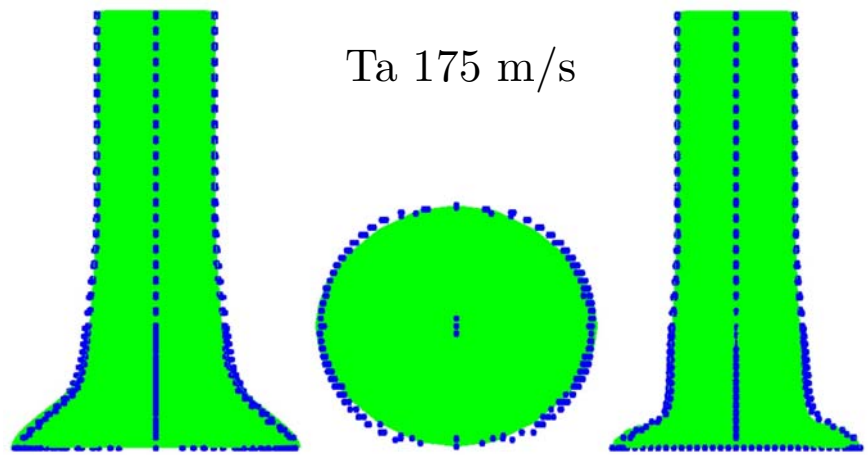




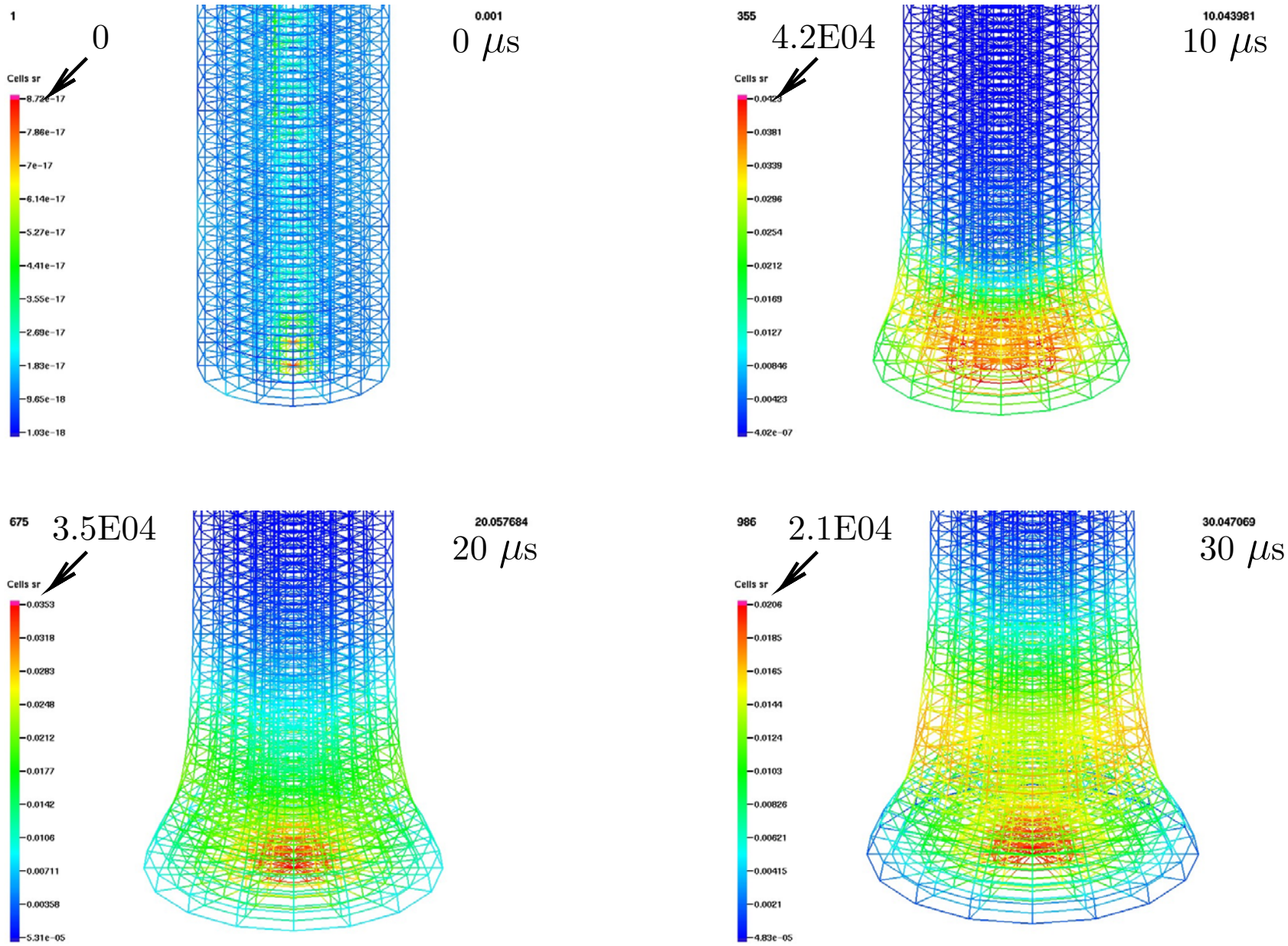
1 Š high-voltage cable;
2 Š massive support;
3 Š liquid explosive tank;
4 Š liquid explosive;
5 Š projectile;
6 Š perforated crusher of AMZ alloy;
7 Š transition device;
8 Š sliding bush;
9 Š loading (measuring) rod;
10 Š sample in the fixed block;
11 Š support (measuring) rod;
12 Š fixed stop;
T1, T2, T3, T4 Š strain sensors.





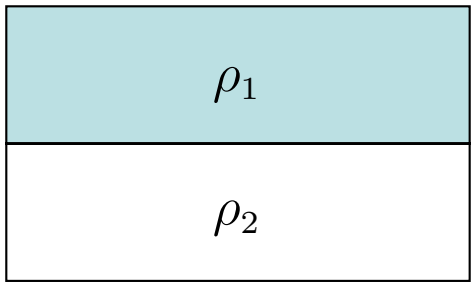


Strain Rate in Taylor Cylinder

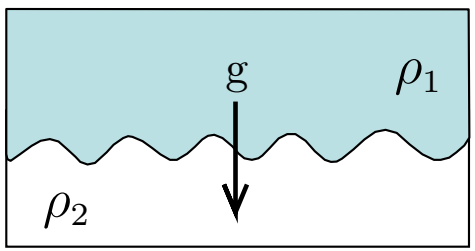


Perturbation Method

Classical Rayleigh-Taylor Instability



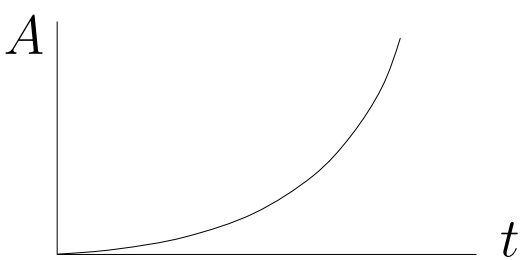
$\rho_1 > \rho_2$



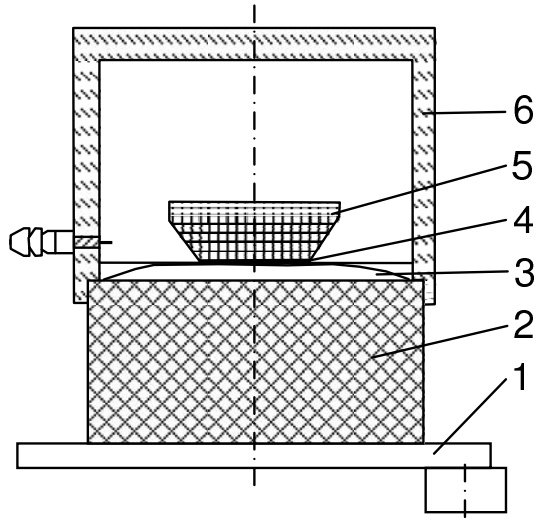
$$A(t) = A_0 e^{\sigma t}$$

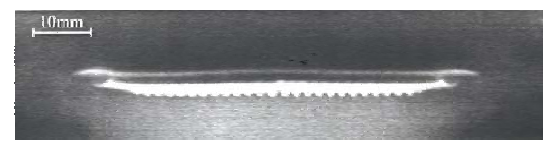
$$\sigma = \sigma(k, a, A_N)$$

$$k = \frac{2\pi}{\lambda} \qquad A_N = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$



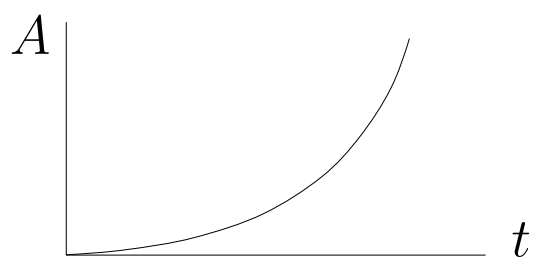
Perturbation Method



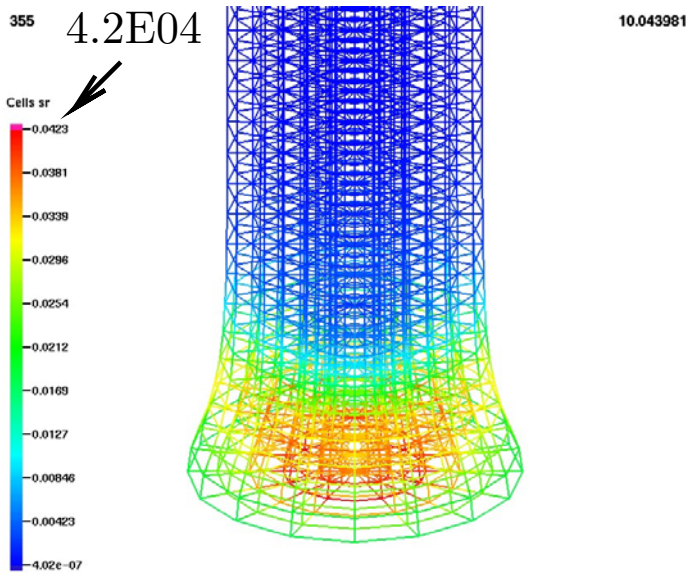


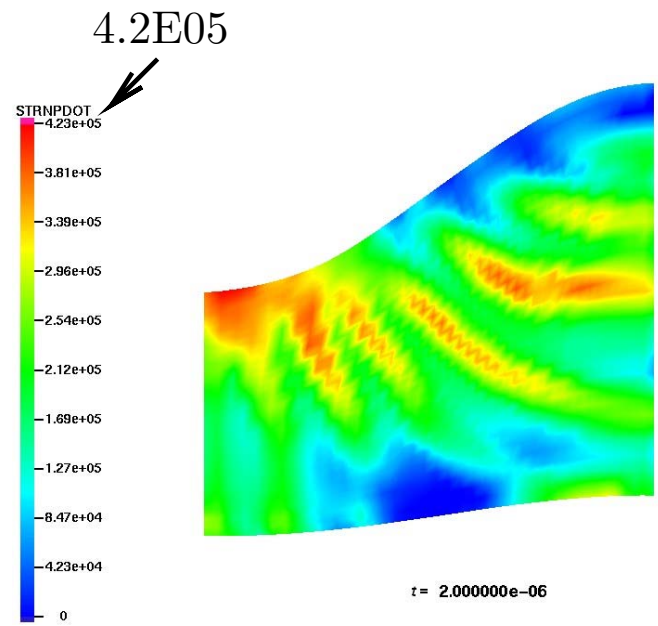
$$A(t) = A_0 e^{\sigma t}$$

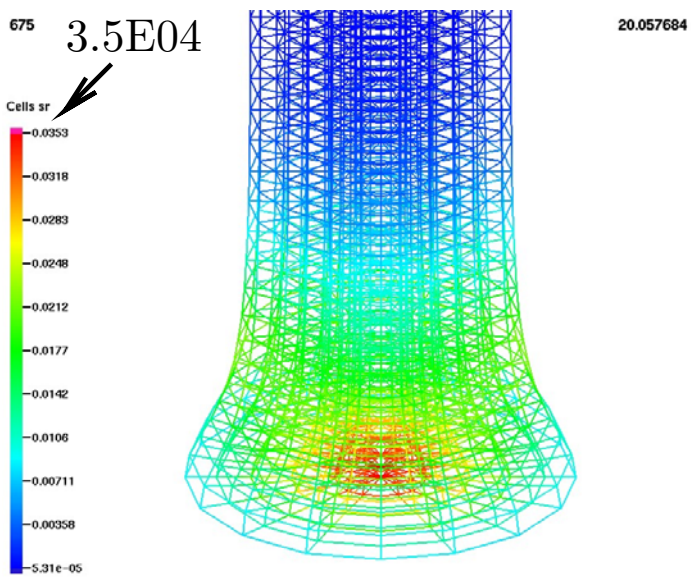
$$\sigma = \sigma(k, a, A_N, G, Y)$$

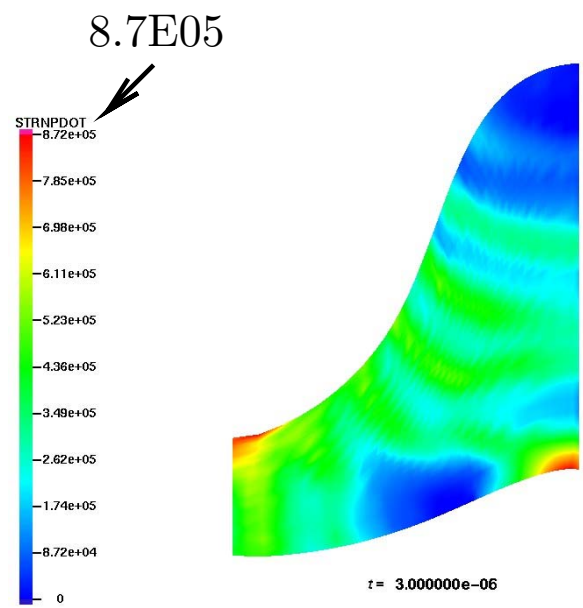


Taylor Cylinder vs Perturbation Method

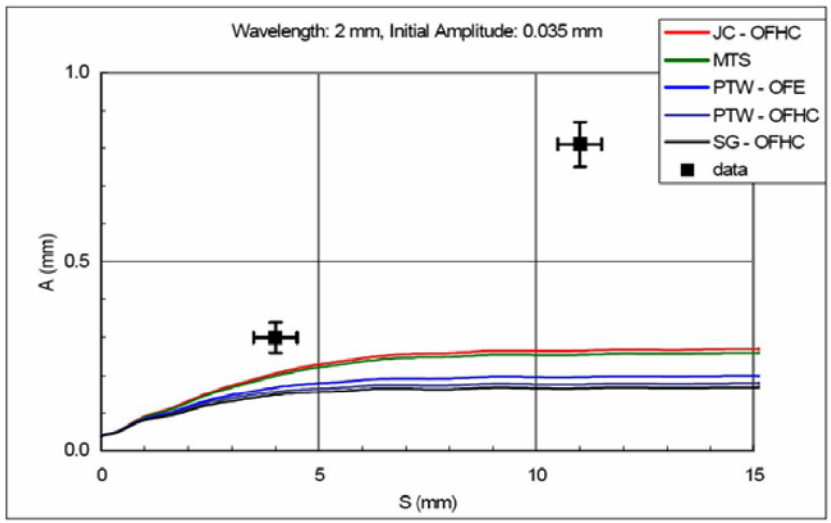


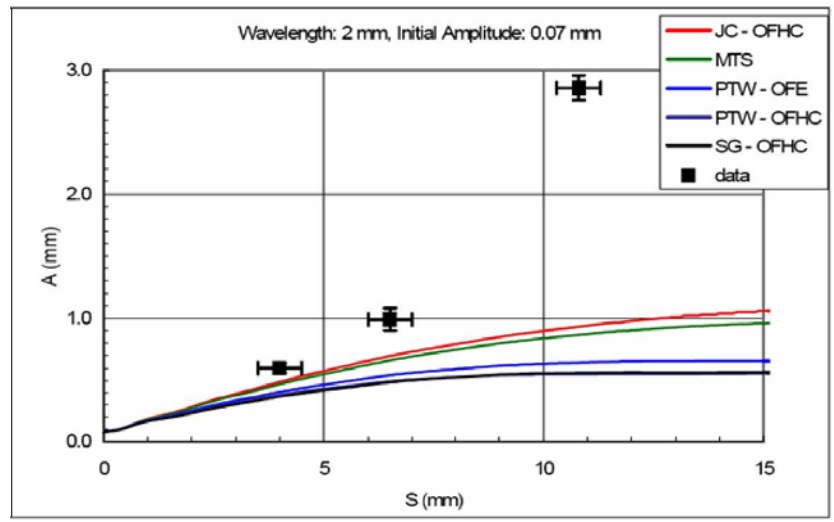




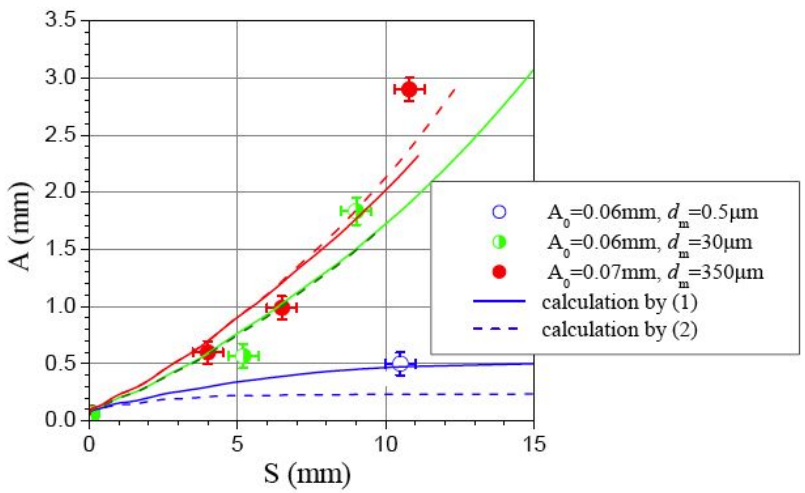


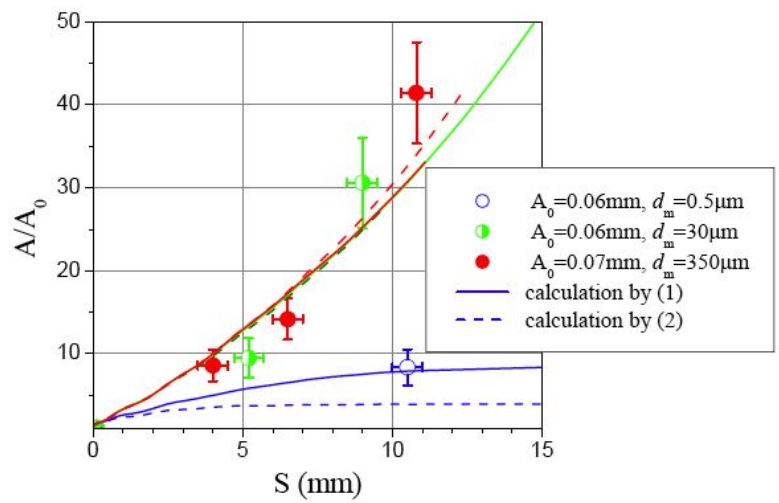
Comparison of Prediction to Data (LANL Baseline Models)



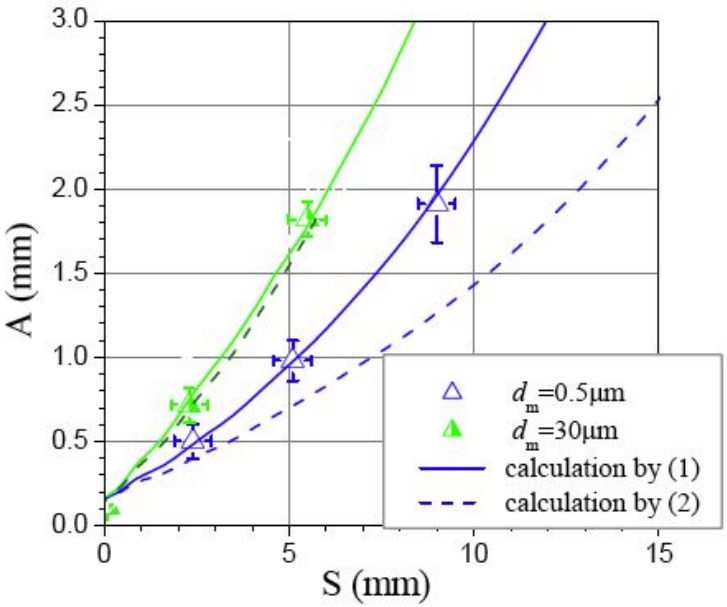


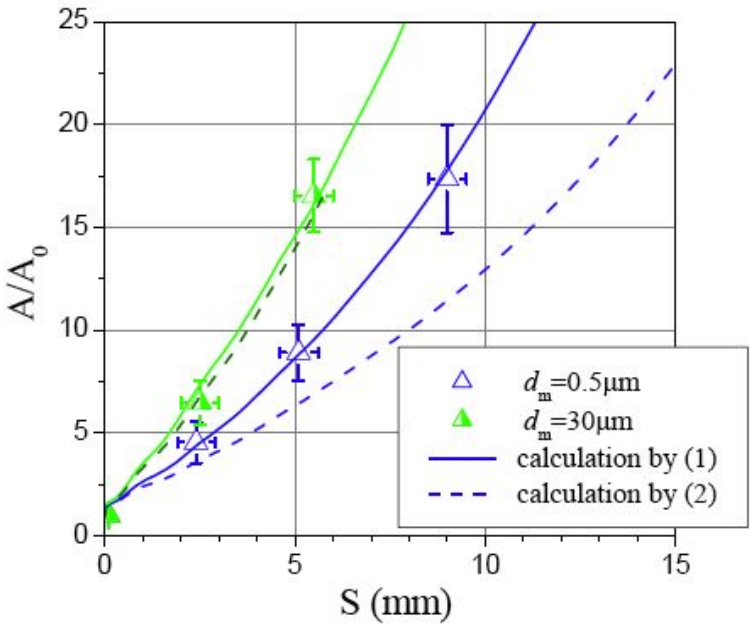
Comparison of Prediction to Data (VNIIEF Model)
$\lambda = 2$ mm, $A_0 = 0.07$ mm



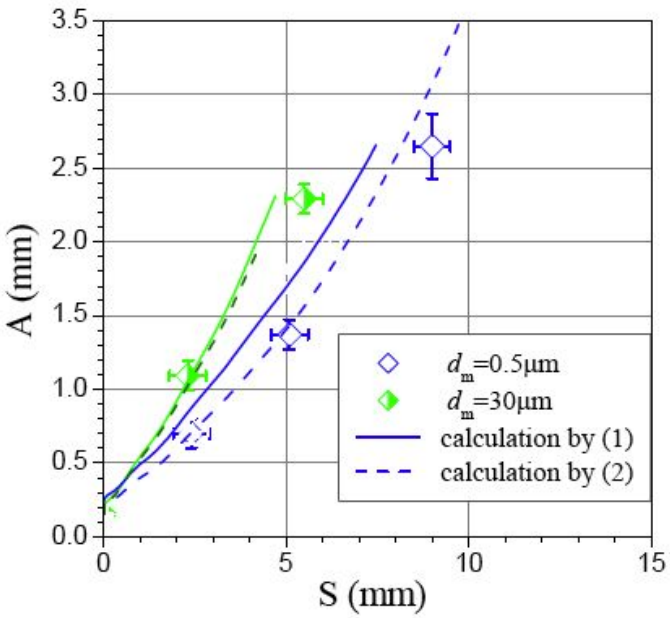


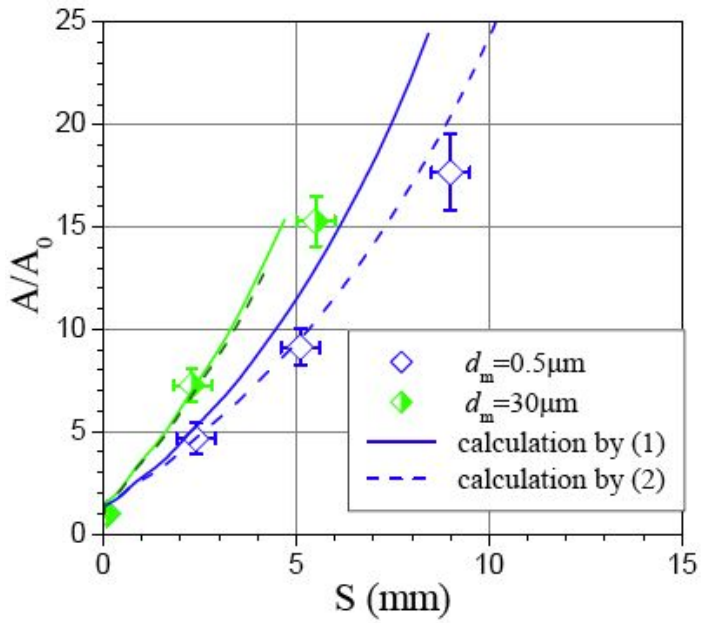
Comparison of Prediction to Data (VNIIEF Model)
$\lambda = 2$ mm, $A_0 = 0.11$ mm





Comparison of Prediction to Data (VNIIEF Model)
$\lambda = 2$ mm, $A_0 = 0.15$ mm





Johnson-Cook

$$\sigma = \left[\sigma_{\text{o}} + B\left(\epsilon^p\right)^n\right]\left[1 + C\,\ln\left(\dot{\epsilon}^p\right)\right]\left[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right]$$

Five input parameters: σ_{o}, B, n, C, T_r, T_m, and m

Steinberg-Guinan

$$G = G_0\left\{1 + A\frac{P}{\eta^{1/3}} - B(T - 300)\right\}$$

$$\sigma = Y_0\left\{1 + \beta(\epsilon_p + \epsilon_i)\right\}^n\left\{1 + A\frac{P}{\eta^{1/3}} - B(T - 300)\right\}$$

$$A = \frac{1}{G_0}\frac{dG}{dP} \qquad B = \frac{1}{G_0}\frac{dG}{dT} \qquad Y_0\left\{1 + \beta(\epsilon_p + \epsilon_i)\right\}^n \leq Y_{\max}$$

$$T_m = T_{m0}\exp\left\{2a(1 - 1/\eta)\right\}\eta^{2(\gamma_0 - a - 1/d)}$$

Ten input parameters: G_0, Y_0, A, B, β, ϵ_i, $Y_{\max}$, n, a, γ_0

MTS

$$\mu = b_0 - \frac{b_1}{\left(e^{\frac{b_2}{T}} - 1\right)} \qquad \widehat{\sigma}_s = \widehat{\sigma}_{s0}\left(\frac{\dot{\epsilon}}{\dot{\epsilon}_{s0}}\right)^{\frac{kT}{\mu A b^3}}$$

$$X_i = \frac{\widehat{\sigma}_i}{\widehat{\sigma}_s} \qquad F(X_i) = \frac{\tanh\left(\alpha X_i\right)}{\tanh \alpha}$$

$$\theta_0 = a_0 + a_1 \ \ln \dot{\epsilon} + a_2 \sqrt{\dot{\epsilon}}$$

$$\frac{\partial \widehat{\sigma}_i}{\partial \epsilon} = \theta_0 \left[1 - F(X_i)\right] \qquad \Rightarrow \qquad \widehat{\sigma}_i(n+1) = \widehat{\sigma}_i(n) + \Delta\epsilon\, \theta_0 \left[1 - F(X_i)\right]$$

$$S_i = \left[1 - \left(\frac{kT \ln \frac{\dot{\epsilon}_i}{\dot{\epsilon}}}{\mu b^3 g_i}\right)^{\frac{1}{q_i}}\right]^{\frac{1}{p_i}}$$

$$\sigma = \widehat{\sigma}_a + \left(\frac{\mu}{\mu_0}\right) \sum_{i=1}^{N} \widehat{\sigma}_i \, S_i$$

Thirteen plus 5N input parameters: N, $\widehat{\sigma}_a$, $\dot{\epsilon}_i$, g_i, p_i, q_i, $\widehat{\sigma}_{i_0}$, b_0, b_1, b_2, $\widehat{\sigma}_{s0}$, $\dot{\epsilon}_{s0}$, A, b, a_0, a_1, a_2, α.

PTW

$$G = G_0 \left(1 - \alpha \widehat{T}\right) \qquad \dot{\xi} = \frac{1}{2}\left(\frac{4\pi\rho}{3M}\right)^{\frac{1}{3}} \sqrt{\frac{G}{\rho}}$$

$$\widehat{\tau}_s = \max\left\{S_0 - (S_0 - S_\infty)\,\mathrm{erf}\left[\kappa \widehat{T} \ln\left(\frac{\gamma\dot{\xi}}{\dot{\psi}}\right)\right],\ S_0\left(\frac{\dot{\psi}}{\gamma\dot{\xi}}\right)^{\beta}\right\}$$

$$\widehat{\tau}_y = \max\left\{y_0 - (y_0 - y_\infty)\,\mathrm{erf}\left[\kappa \widehat{T} \ln\left(\frac{\gamma\dot{\xi}}{\dot{\psi}}\right)\right],\ \min\left[y_1\left(\frac{\dot{\psi}}{\gamma\dot{\xi}}\right)^{y_2},\ S_0\left(\frac{\dot{\psi}}{\gamma\dot{\xi}}\right)^{\beta}\right]\right\}$$

$$\widehat{\tau} = \widehat{\tau}_s + \frac{1}{p}(s_0 - \widehat{\tau}_y) \ *$$

$$\ln\left[1 - \left[1 - \exp\left(-p\frac{\widehat{\tau}_s - \widehat{\tau}_y}{S_0 - \widehat{\tau}_y}\right)\right] \exp\left\{-\frac{p\theta_0\psi}{(S_0 - \widehat{\tau}_y)\left[\exp\left(p\frac{\widehat{\tau}_s - \widehat{\tau}_y}{S_0 - \widehat{\tau}_y}\right) - 1\right]}\right\}\right]$$

$$\sigma = 2\,\widehat{\tau}\,G$$

Fifteen input parameters: p, θ_0, κ, γ, α, G_0, T_m, M, S_0, S_∞, y_0, y_∞, β, y_1, y_2

VNIIEF Models -1

Relaxation Model

$$\dot{Y}_d = 3G\dot{\varepsilon}_i - \frac{Y_d - Y_s}{\tau} \qquad (\sigma_i \geq Y_s)$$

$$Y_s = Y_0 \left[1 + a_0 \left(1 - \overline{T}^k\right) \left(1 - \exp\left(-n\varepsilon_i^p\right)\right)\right] \left[1 - \overline{T}\right] \left[1 + \alpha_0 P\right]$$

$$T_m = T_{om} \left(\frac{\delta_m}{\delta_{0m}}\right)^{2\left(\Gamma_\infty - \frac{1}{3}\right)} \exp\left[\frac{2\left(\Gamma_0 - \Gamma_\infty\right)}{M} \left(\frac{1}{(\delta_{om})^M} - \frac{1}{(\delta_m)^M}\right)\right]$$

$$\tau = \frac{\tau_0 \left(1 - \overline{T}\right)}{\left(1 + \frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_{i0}}\right)^{0.5}}$$

Special Cases:

(a) if $\tau \, \frac{d \ln Y_d}{dt} << 1$ the model simplifies to:

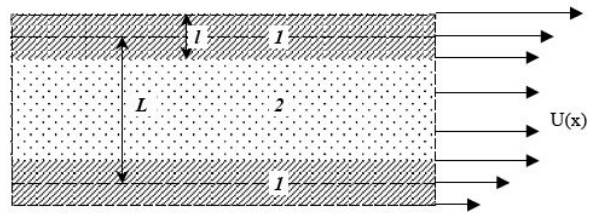
$$Y_d = Y_s + 3\, g\, \tau\, \dot{\epsilon}_i$$

(b) if $\dot{\epsilon}_i = 0$:

$$Y_d = Y_s$$

VNIIEF Models -2

Heterogeneous Deformation Model (Two-Temperature Model)



Condition for transition to heterogeneous deformation:

$$\left(\frac{\partial Y_s}{\partial \varepsilon_i}\right)_{P,E_T} + \frac{1}{2\rho}\left(\frac{\partial\left(Y_s + 3G\dot{\varepsilon}_i\tau\right)^2}{\partial E_T}\right)_{P,\dot{\varepsilon}_i} \geq 0$$

$$\dot{Y}_d = 3G\dot{\varepsilon}_i - \frac{Y_d - \widetilde{Y}_s}{\tau}\left(1 + \frac{l}{L}\left(\frac{G\,\tau}{\widetilde{G}\,\widetilde{\tau}} - 1\right)\right)$$

$$Y_s = Y_0\left[1 + a_0\left(1 - \overline{T}^{\,k}\right)\left(1 - \exp\left(-n\varepsilon_i^p\right)\right)\right]\left[1 - \overline{T}\right]\left[1 + \alpha_0 P\right]$$

$$L = \frac{\chi}{C_1}\left(\frac{\eta\,\rho\,\left(E_m - E_{T0}\right)}{\sigma_i\varepsilon_i}\right)^2\left(1 + \frac{\tau_f\,C_1}{L}\right)^2$$

$$l = \sqrt{\frac{\chi\,L}{C_1}}\left(\frac{\eta\,\rho\,\left(E_m - E_{T0}\right)}{\sigma_i\varepsilon_i}\right)$$

Conclusions

- Typically employed methodology results in model validation to strain rates of $10^4 \sim 10^5$
- Some problems of interest see higher strain rates
- Application of models validated using the typically employed methodology to such problems results in extrapolation beyond the validated strain rate regime
- Perturbation Method enables validation of models to strain rates of $10^6 \sim 10^7$
- Such validation will improve our predictive capability