The Effect of Viscosity, Dispersion, and Phase Transition Kinetics on the Parameters of Rarefaction Shock Waves for Non-Convex Equations of State

Yu.A.Bondarenko, V.N.Sofronov

Russian Federal Nuclear Center – VNIIEF, 37 Mir Ave., Sarov, Nizhni Novgorod region, Russia

It has been analytically proved that consideration of various physical processes, such as viscosity, dispersion, or kinetics of phase transitions, leads to different parameters of rarefaction shock waves emerging in materials having non-convex equations of state. One-dimensional simulations confirm these results. Hence, the problem of selecting a single and "correct" rarefaction shock wave should be solved at the level of physical model selection and depends on major physical processes to be ignored, when writing the ideal gas dynamics equations.

Introduction

Materials with abnormal thermodynamic properties rouse curiosity of researchers in gas dynamics, if Bethe-Weil conditions are violated, in particular, if adiabatic curve in plane *(P,V)* has non-convex sections. In such cases, a compressed material unloading causes rarefaction shock waves (Zel'dovich and Raiser, 1963). For the sake of simplicity, let us not go beyond the barotropic equation of state in which pressure depends on the specific volume alone. Assume that the equation of state $p=P_0(v)$ is smooth enough, strictly monotonous and has only one section of "reverse" convexity

$$\begin{aligned} &\frac{d^2V_0(p)}{dp^2} > 0, \quad \text{если} \quad p < p_{min} \quad \text{или} \quad p > p_{max}; \\ &\frac{d^2V_0(p)}{dp^2} < 0, \quad \text{если} \quad p_{min} < p < p_{max}; \end{aligned} \qquad [1]$$

For any continuous monotonous initial profile $p_0(m)=p(t,m)|_{t=0}$, $u_0(m)=u(t,m)|_{t=0}$, $p(t,m)|_{m=0}=p_-<p_{min}$, $p(t,m)|_{m=+\infty}=p_+>p_{max}$, which unloads leftwards, characteristics within the pressure range $p_{min}<p<p_{max}$ will inevitably intersect after some time and this will lead to a rarefaction shock wave with parameters $(p_1,v_1,u_1)\rightarrow(p_2,v_2,u_2)$, where $p_2<p_1$, $v_2>v_1$, $u_2<u_1$. The rarefaction shock wave parameters in case of barotropic EOS meet two Renckin-Hugoniot requirements and, hence, the mass velocity of the rarefaction shock wave is

$$\sigma = \sqrt{\frac{p_1 - p_2}{V_0(p_2) - V_0(p_1)}} \qquad [2]$$

All the profiles should approach with time the self-similar solution of the Riemann problem. The rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ in the limit self-similar solution should meet the requirements

$$p_2 \le p_{min} < p_{max} \le p_1 \qquad [3]$$

$$\left(s(p_2)\right)^2 \equiv \left(-\frac{dV_0(p)}{dp}\bigg|_{p=p_2}\right)^{-1} \le \sigma^2 \equiv \frac{p_1-p_2}{V_0(p_2)-V_0(p_1)} \le \left(s(p_1)\right)^2 \equiv \left(-\frac{dV_0(p)}{dp}\bigg|_{p=p_1}\right)^{-1} \qquad [4]$$

In plane (P,V), the rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ meeting Renckin-Hugoniot requirements is represented by Rayleigh-Michelson straight-line segment

$$V_{RM}(p) = \frac{(p_1-p)V_0(p_2)+(p-p_2)V_0(p_1)}{(p_1-p_2)} \qquad [5]$$

Fig.1 shows examples of positions of Rayleigh-Michelson straight-line segment satisfying conditions 3 and 4. There is a self-similar solution for each rarefaction shock wave of such a type. To select a single solution from this variety, additional selection criteria are required. The requirement of an admissible shock wave is usually used, for the barotropic equation of state it has the following form: an impact transition $(p_1,v_1,u_1)\rightarrow(p_2,v_2,u_2)$ is considered admissible, if there is a classic solution to the gas dynamic equation system for viscous gas with any positive coefficient of viscosity $\mu>0$ that connects points (p_1,v_1,u_1) and (p_2,v_2,u_2). Such impact transition is said to be admissible by the vanishing viscosity method.

Galin (1959) proved that the impact transition $(p_1,v_1)\rightarrow(p_2,v_2)$ in the form of a rarefaction shock wave is admissible, if adiabatic curve $v=V_0(p)$is in plane (P,V) is above Rayleigh-Michelson straight line $v=V_{RM}(p)$. In the variety of cases in Fig.1 the criterion of admissibility by the vanishing viscosity method allows determining a single rarefaction shock wave which Rayleigh-Michelson straight-line segment is tangent to the adiabatic curve $v=V(p)$ at both its end points (p_1,v_1) and (p_2,v_2). In so doing, there is no need in using the entropy requirement of Lax essential non-linearity (Lax, 1957) and the shock wave evolution requirement (Gelfand, 1959) to select the single solution. The requirement of shock wave admissibility is of a higher priority.

Such definition of admissibility of a rarefaction shock wave is based on a common assumption that the major physical process to be omitted with the use of the ideal gas dynamics equations is viscosity. In this paper we consider other definitions of admissible rarefaction shock waves and show that parameters of rarefaction shock waves in the self-similar Riemann problem depends on the physical processes that determine the structure of smeared shock waves.

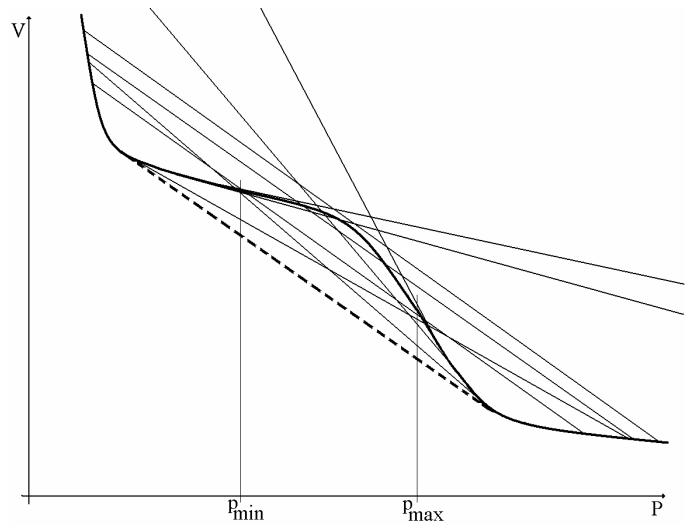

Figure 1 – Possible variants of positions of Rayleigh-Michelson straight-line segment $v=V_{RM}(p)$ for rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ meeting the necessary requirements (3) and (4). The dotted line represents the single rarefaction shock wave admissible by the vanishing viscosity method.

The Method of Vanishing Normal Dispersion and Rarefaction Shock Waves in the Self-Similar Riemann Problem

Consider the small normal dispersion ($\eta = const > 0$, $\eta\rightarrow+0$) in 1D gas dynamics equations in Lagrangian variables

$$\frac{dv}{dt} - \frac{\partial u}{\partial m} = 0 \qquad [6]$$

$$\frac{du}{dt} + \frac{\partial p}{\partial m} - \eta \frac{\partial^3 v}{dt^2 \partial m} = 0 \qquad [7]$$

We say that the rarefaction shock wave $(p_1,v_1,u_1)\rightarrow(p_2,v_2,u_2)$ is admissible by normal dispersion (or admissible by the method of normal dispersion), if there is a classic solution to the system of gas dynamics equations 5 and 7 with normal dispersion and with any positive value of dispersion coefficient $\eta>0$ that connects points (p_1,v_1,u_1) and (p_2,v_2,u_2).

Consider the structure of time-independent shock wave and determine conditions under which equations 6 and 7 have continuous solutions of the form

$$\vec{U}(t,m) = \vec{U}(\mu); \quad \vec{U} = (u,P,v); \quad \mu = m - \sigma \cdot t; \quad \sigma = const > 0 \qquad [8]$$

with boundary conditions at infinity

$$\vec{U}(\mu) \rightarrow \vec{U}_2, \ \frac{\partial \vec{U}}{\partial \mu} \rightarrow 0 \ \ at \ \ \mu \rightarrow -\infty \qquad [9]$$

$$\vec{U}(\mu) \to \vec{U}_1, \quad \frac{\partial \vec{U}}{\partial \mu} \to 0 \ \ при \ \ \mu \to \infty \qquad [10]$$

Substitute the solution in the form of a progressive wave (equation 8) into equations 6 and 7 and obtain upon velocity elimination that

$$\sigma^2 \eta \cdot \frac{d^2 v}{d\mu^2} = P_0(v) - p_1 + \sigma^2 \cdot (v - v_1) = F(v) \qquad [11]$$

where σ –is the mass velocity of shock discontinuity propagation, equation 2. Equation 11 can be easily integrated and it can be easily checked that the integral curve connecting points (p_1,v_1) and (p_2,v_2) in plane (P,V) exists, if and only if function

$$\Phi(v) = \int_{v_1}^{v} \left[P_0(v) - \frac{p_1 \cdot (v_2 - v) + p_2 \cdot (v - v_1)}{v_2 - v_1} \right] dv \qquad [12]$$

is positive for all values of v within the range $v_1<v<v_2$ and vanishes at the end points of this range. Hence, we obtain the following (Bondarenko et al., 2003).

Theorem 1. In the self-similar Riemann problem with non-convex equation of state there is a rarefaction shock wave admissible by the method of vanishing normal dispersion. This rarefaction shock wave is the single one uniquely determined by the following features. First, in plane (P,V) the segment of Rayleigh-Michelson straight line $v=V_{RM}(p)$ of the rarefaction shock wave $(p_1,v_1)\to(p_2,v_2)$ is tangent to the equation of state $v=V_0(p)$ at its end point (v_1,p_1) and, secondly, the Rayleigh-Michelson straight line intersects the adiabatic curve $v=V_0(p)$ at an intermediate point and at the end point (p_2,v_2), so that the integral in equation 12 for $v=v_2$ equals zero.

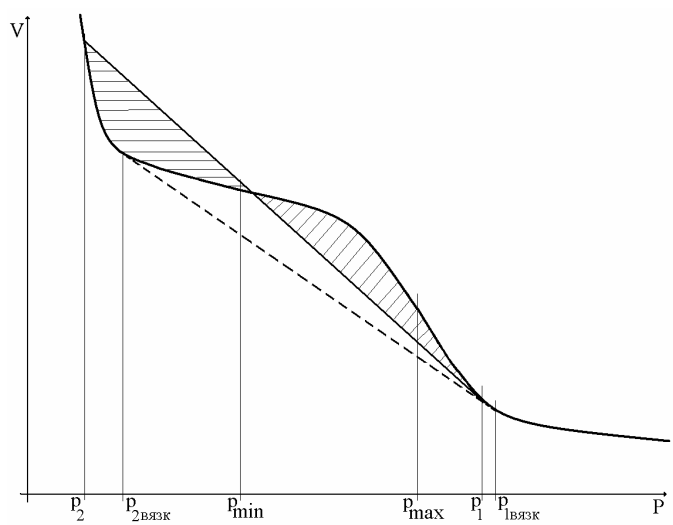

Figure 2 – The adiabatic curve with the non-convex equation of state and Rayleigh-Michelson straight lines of the rarefaction shock wave admissible by the method of vanishing normal dispersion ($p_1\to p_2$) and the method of vanishing viscosity ($p_{1вязк}\to p_{2вязк}$).

Such admissible solutions are sketched in Fig.2. At point (p_1,v_1) Rayleugh-Michelson straight line $v=V_{RM}(p)$ should be tangent to adiabatic curve $v=V_0(p)$ and the areas of two shaded curvilinear triangles should be equal. The rarefaction shock wave obtained using the method of vanishing

normal dispersion always differs from that obtained using the method of vanishing viscosity. Event velocities of such rarefaction shock waves are always different.

The Method of Artificial Interphase Kinetics and Rarefaction Shock Waves in the Self-Similar Riemann Problem

Artificial interphase kinetics can be also used instead of viscosity and dispersion for smearing rarefaction shock waves. For this purpose, use the two-phase model similar to that described in the paper by Akhmodeyev et al. (1984), with natural generalizations and simplifications adopted in this paper. It is assumed that a material consists of two mixed phases with convex equations of state $v=V_1(p)$ and $v=V_2(p)$ for each of the pure phases, the equilibrium equation of state

$$v=V_0(p)\equiv\Theta_0(p)\cdot V_1(p)+(1-\Theta_0(p))\cdot V_2(p) \qquad [13]$$

is strictly monotonous, however, it has one non-empty range of non-convexity, where $\Theta_0(p)$ is a strictly monotonous equilibrium concentration of the first phase. Assume that requirement 1 and requirements

$$\frac{d^2V_k(p)}{dp^2}>0, \quad k=1,2 \qquad [14]$$

$$\frac{1}{\left(s_k(p)\right)^2}=-\frac{dV_k(p)}{dp}>0, \quad k=0,1,2 \qquad [15]$$

$$\lim_{p\to+\infty}\Theta_0(p)=1; \qquad \lim_{p\to+0}\Theta_0(p)=0 \qquad [16]$$

$$0\le\frac{d\Theta_0(p)}{dp}\le const<+\infty, \qquad \frac{d\Theta_0(p)}{dp}>0 \quad npu \quad 0<\Theta_0(p)<1 \qquad [17]$$

are met. In the artificial interphase kinetics method the non-equilibrium equation of state

$$v=V(p,\Theta)\equiv\Theta\cdot V_1(p)+(1-\Theta)\cdot V_2(p) \qquad [18]$$

is used and the concentration $\Theta(t,m)$ of the first phase is described by the non-equilibrium equation of the phase transition kinetics

$$\tau\frac{d\Theta}{dt}=-\left(\Theta-\Theta_0(p)\right)\cdot\omega(\Theta,p) \qquad [19]$$

with positive time of phase relaxation $\tau=const\to+0$ and positive function $\omega(\Theta,p)$ continuous over the set of arguments, which describes the dependence of the "rate" of interphase relaxation on pressure and phase concentrations. Viscosity and dispersion are not taken into consideration.

Criteria of presence of rarefaction shock waves smeared by the artificial interphase kinetics and progressing with constant-velocities

First, we study the issue of presence of smeared rarefaction shock waves progressing with constant velocities. For this purpose, the solution in the form of a progressive shock wave 8 is substituted into equations 20 and 21 of 1D Lagrangian gas dynamics

$$\frac{dv}{dt} - \frac{\partial u}{\partial m} = 0 \qquad [20]$$

$$\frac{du}{dt} + \frac{\partial p}{\partial m} = 0 \qquad [21]$$

added by equation of state 18 and kinetic equation 19.

Assume that gas behind the rarefaction shock wave is in idle state. Find continuous and continuously differentiated solutions of such a kind that they achieve constant values at infinity, i.e. satisfy edge conditions 9 and 10 at infinity and also satisfy the conditions below

$$\Theta(\mu) \to \Theta_1 = \Theta_0(p_1) \ npu \ \mu \to +\infty \qquad [22]$$

$$\Theta(\mu) \to \Theta_2 = \Theta_0(p_2) \ npu \ \mu \to -\infty \qquad [23]$$

$$\frac{\partial \Theta}{\partial \mu} \to 0 \ \ npu \ \ \mu \to \pm\infty \qquad [24]$$

As a result, we obtain the system of ordinary differential equations

$$\sigma \frac{dv}{d\mu} + \frac{du}{d\mu} = 0 \qquad [25]$$

$$\sigma \frac{du}{d\mu} - \frac{dp}{d\mu} = 0 \qquad [26]$$

$$\tau\sigma \frac{d\Theta}{d\mu} = \left(\Theta - \Theta_0(p)\right) \cdot \omega(\Theta, p) \qquad [27]$$

Integration of equations 25 and 26 leads to

$$\Theta = \vartheta(p) \equiv \frac{V_2(p) - V_{RM}(p)}{V_2(p) - V_1(p)} \qquad [28]$$

Use equation 28 to eliminate the first phase concentration from equation 27 and obtain one ordinary differential equation, namely

$$\frac{\vartheta'(p)}{\psi(p) \cdot \omega(\vartheta(p), p)} \cdot \frac{dp}{d\mu} = \frac{1}{\tau\sigma}, \qquad [29]$$

which can be *pro forma* (and quite easily) integrated in quadratures. The solution to equation 29 must satisfy edge conditions 9 and 10 at infinity, the other edge conditions at infinity will be satisfied automatically. For existence of a monotonous solution satisfying the conditions above at infinity the inequality

$$\psi(p) = \Theta - \Theta_0(p) = \frac{V_0(p) - V_{RM}(p)}{V_2(p) - V_1(p)} > 0, \quad \forall p : p_2 < p < p_1 \qquad [30]$$

must be valid.

The following result can be easily proved.

Conjecture. The rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ smeared using the artificial interphase kinetics exists, if and only if the following conditions are satisfied simultaneously:

(i) function $\psi(p)=\dfrac{V_0(p)-V_{RM}(p)}{V_2(p)-V_1(p)}$ is strictly positive for all values of p within the range $p_2<p<p_1$;

(ii) function $\vartheta'(p)=\dfrac{d}{dp}\left(\dfrac{V_2(p)-V_{RM}(p)}{V_2(p)-V_1(p)}\right)$ is strictly positive for all values of p within the range $p_2<p<p_1$;

(iii) function $f(p)=\vartheta'(p)/\psi(p)$ is continuous everywhere within the range $p_2<p<p_1$;

(iv) function $1/f(p)\rightarrow 0$ at $p\rightarrow p_1-0$;

(v) function $1/f(p)\rightarrow 0$ at $p\rightarrow p_2+0$.□

Further analysis will be performed separately for two cases, when the equations of state of pure phases $v=V_1(p)$ and $v=V_2(p)$ do not intersect and when they intersect at one point only.

Extreme rarefaction shock waves in the method of artificial interphase kinetics in case of not intersected equations of state of pure phases.

Consider a case of not intersecting equations of state of two pure phases, $v=V_1(p)$ and $v=V_2(p)$:

$$V_2(p)>V_1(p);\quad P_2(v)>P_1(v). \qquad [31]$$

If the requirement 31 of not intersecting pure phases is met, the necessary requirement 30 concerning the existence of the smeared rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ becomes

$$V_{RM}(p)<V_0(p),\quad \forall p: p_2<p<p_1 \qquad [32]$$

Theorem 2. Let the equations of state of two pure phases $v=V_1(p)$ and $v=V_2(p)$ be not intersected (i.e. the requirement 31 is met) and assumptions 1 and 14-17 are valid. Then, there is a rarefaction shock wave obtained using the method of artificial interphase kinetics. This is a single rarefaction wave , which is uniquely determined by the following feature. In plane (P,V) the Rayleigh-Michelson straight-line segment $v=V_{RM}(p)$ of the rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ is tangent to the equilibrium equation of state $v=V_0(p)$ at its end points (v_2,p_2) and (v_1,p_1).□

The paper by Bondarenko and Sofronov (2004) gives the complete proof of Theorems 2 and 3 (it is given below).

The parameters of the rarefaction shock wave obtained using the method of artificial interphase kinetics are the same as the parameters of the rarefaction wave obtained for the equilibrium equation of state $v=V_0(p)$ with the method of vanishing viscosity.

Extreme rarefaction shock waves in the method of artificial interphase kinetics for intersected equations of state of pure phases.

Now consider the method of artificial interphase kinetics for a case, when the equations of state of two pure phases, $v=V_1(p)$ and $v=V_2(p)$ intersect at a single point (p_3,v_3), $p_1>p_3>p_2$ and assume that

$$\begin{aligned} &P_2(v) > P_1(v) \;\; npu \;\; v < v_3; \\ &P_2(v) < P_1(v) \;\; npu \;\; v > v_3, \end{aligned} \qquad [33]$$

$$\left(s_1(p_3)\right)^2 = -\left(\frac{dV_1(p)}{dp}\right)^{-1}\Bigg|_{p=p_3} < \left(s_2(p_3)\right)^2 = -\left(\frac{dV_2(p)}{dp}\right)^{-1}\Bigg|_{p=p_3} \qquad [34]$$

Basing on the necessary condition (equation 30) of existence of the rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ smeared by the artificial interphase kinetics and the condition (equation 33) of intersection of pure phases, we obtain the following necessary conditions for Rayleigh-Michelson straight-line segment positioning

$$\begin{aligned} &V_{RM}(p) > V_0(p), \;\; p < p_3; \\ &V_{RM}(p) < V_0(p), \;\; p > p_3. \end{aligned} \qquad [35]$$

It follows from this, in particular, that Rayleigh-Michelson straight line $v=V_{RM}(p)$ should pass through the point of intersection of pure phases, (p_3,v_3), that is $V_{RM}(p_3) = v_3$.

Theorem 3. Let the equations of state of two pure phases $v=V_1(p)$ and $v=V_2(p)$ intersect at a single point (p_3,v_3) and the assumptions 1, 14÷17 and 33÷34 be valid. Then, there is a rarefaction shock wave obtained using the method of artificial interphase kinetics in the self-similar Riemann problem. This rarefaction shock wave is a single one and it is uniquely determined by the following feature. In plane (P,V) the Rayleigh-Michelson straight-line segment $v=V_{RM}(p)$ of the rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ passes through the phase intersection point (p_3,v_3), is tangent (from below) to the equilibrium equation of state $v=V_0(p)$ at one of its end points, (v_1,p_1) and intersects (from above and with no contacts) the equilibrium equation of state at the second of its end points, (p_2,v_2).□

In case of intersected equations of state of pure phases, the parameters of the rarefaction shock wave obtained using the method of artificial interphase kinetics significantly differ from that of the rarefaction wave obtained for the equilibrium equation of state $v=V_0(p)$ using the method of vanishing viscosity and method of vanishing dispersion. This statement will be proved by computation results.

Results of 1D Computations for Self-Similar Riemann Problems With Rarefaction Shock Waves

It is well known (Turkel, 1974) that errors of homogenous difference schemes of end-to end computations for 1D gas dynamics are the sum of approximation (and/or artificial) viscosity and approximation dispersion errors. In the absence of approximation dissipation (viscosity) in a difference scheme in computations without artificial viscosity, approximation dispersion of the second infinitesimal order will be a predominant mechanism of rarefaction shock wave smearing. With the use of linear artificial viscosity of the first infinitesimal order on rarefaction waves this viscosity suppresses the effect of approximation dispersion. In order to verify the statement, 1D gas

dynamic computations by UP program (Batalov et al., 1978) using the difference scheme "krest" (cross) with controllable dissipation were carried out.

Problem 1

Problem 1 considers a barotropic equation of state with piecewise analytical representation of the sound speed for each density range $\rho_{k-1} < \rho < \rho_k$ of the form $c = b_k \cdot \rho^{\gamma_k}$, $b_1=4.0$, $b_2=1.0$, $\gamma_1=\gamma_2=1.0$. For such equation of state one can construct in quadratures a self-similar solution for a centered rarefaction wave. In general, the self-similar solution can consist of the following three types of solutions "glued together": non-trivial self-similar solution, trivial solution (a flow of constant density, pressure, and mass velocity values) and impact discontinuity (on which the known Hugoniot relations and requirements of discontinuity admissibility for various regularization ways are met).

The time-independent self-similar pressure profile is shown in Fig.3. Thick lines show the results of analytical solution, thin lines represent the results of numerical computations. Thin lines are overlapped with thick lines almost everywhere. Small differences are observed only near strong and weak discontinuities.

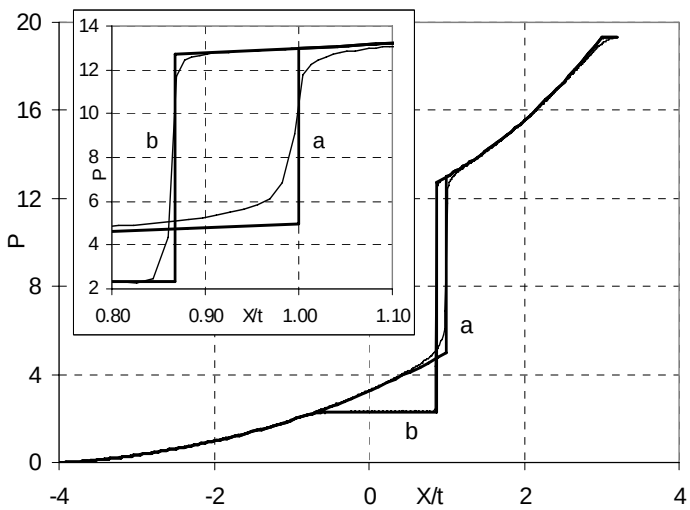

Figure 3 – The pressure profiles for problem 1. Computations with viscosity (a) and dispersion (b).

Problem 2. The mixture of two phases without intersection of state equations of pure phases

Consider a nonequilibrium barotropic model of a two-phase medium. The equation of state of the form $\frac{\rho_0}{\rho_i} = V_i(P) = \left(\frac{a_i}{P - b_i}\right)^{n_i}, i = 1,2$ (Table 1) is given for each pure phase. For the two-phase mixture, use the requirement of volume additivity $\frac{\rho_0}{\rho} = V = \Theta \cdot V_1(P) + (1-\Theta) \cdot V_2(P)$, where Θ is the mass concentration of the first phase. Assume that within the range of pressures $p_{12} < p < p_{21}$ simultaneous phase transitions $1 \to 2$ and $2 \to 1$ are possible. There are no phase transitions outside this pressure range. The kinetic equation of the form

$$\frac{d\Theta}{dt} = j_{21} - j_{12} = \frac{V_2(p)\cdot f_{21}(p)\cdot(1-\Theta) - V_1(p)\cdot f_{12}(p)\cdot\Theta}{\tau}$$

is taken. Here, τ is the interphase relaxation time. The velocity of phase transitions $1 \to 2$ and $2 \to 1$ is determined by the functions f_{12} and f_{21}, which form was taken from the paper by Akhmadeyev et al. (1984):

$$f_{12} = \begin{cases} 1 - exp\left(-\left(\frac{p - p_{12}}{\Delta_{12}}\right)^3\right) & p > p_{12} \\ 0 & p < p_{12} \end{cases} \qquad f_{21} = \begin{cases} 1 - exp\left(-\left(\frac{p_{21} - p}{\Delta_{21}}\right)^3\right) & p < p_{21} \\ 0 & p > p_{21} \end{cases}$$

Fig.3 shows the equations of state of pure phases and the equilibrium mixture of these phases.

Table 1. EOS parameters for problems 2 and 3

Nos.	ρ_0	a_1	b_1	n_1	a_2	b_2	n_2	p_{12}	p_{21}	Δ_{12}	Δ_{21}	τ
4	7.85	6.9523	-6.9523	16	19.845	-23.824	6	12.1	18.6	3.2	3.2	0.04
5	7.85	6.9523	-6.9523	16	0.2424	13.09027	25	13.8	15.45	1.2	1.15	0.04

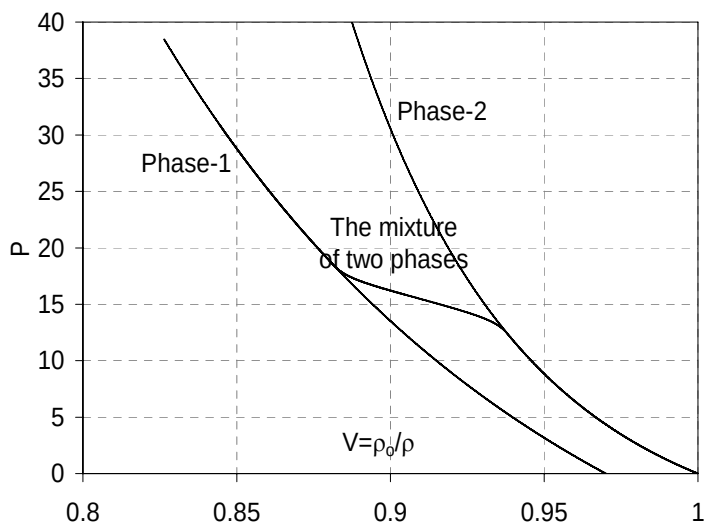

Figure 3 – The two-phase EOS adiabatic curve for problem 2.

The results of comparison between the calculated self-similar pressure profiles for the three regularization variants are shown in Fig.4. The extreme solutions obtained using the method of viscosity and the method of interphase relaxation are the same and differ from the solution with taking account of dispersion. The calculated parameters of rarefaction shock waves meet the requirements of admissibility of discontinuities for the three variants of gas dynamic equation regularization.

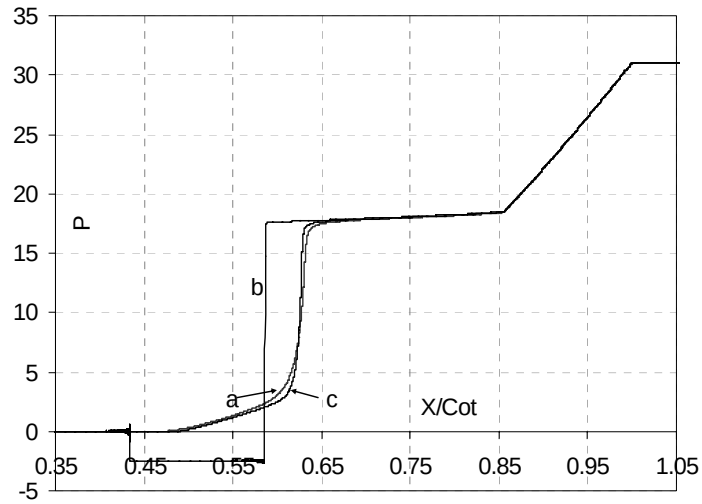

Figure 4 – The pressure profile of the centered rarefaction wave for the equation of state without intersection of phases. Computations with viscosity (a), dispersion (b), and interphase relaxation (c).

Problem 3. The mixture of two phases with intersection of EOSes of the pure phase.

The EOS parameters for the variant with intersection of adiabatic curves of pure phase are given in Table 1. EOSes separately for each phase and the equilibrium EOS for the phase mixture are shown in Fig.5. It is clearly seen that the three equations of state intersect at one point. Fig.6 shows the results of comparison between the calculated self-similar pressure profiles of the rarefaction shock wave in plane for the three regularization variants. The results differ from the earlier presented ones in that each of the three regularization methods gives its own solution differing from the others and this agrees with the results of analytical studies.

The calculated parameters of rarefaction shock waves in (P, v) coordinates are shown in Fig.7. It follows that these parameters meet the requirement of admissibility of discontinuities for the three methods of regularization of gas dynamics equations. Note that for in case of intersection of phases an admissible (from viewpoint interphase relaxation) rarefaction shock wave must go across the point of intersection of phases in (P, v) coordinates and be in contact (from below) with the equilibrium equation of state $P = P(v)$. The enlarged fragment in Fig.7 shows that the calculated shock wave meets the requirements above.

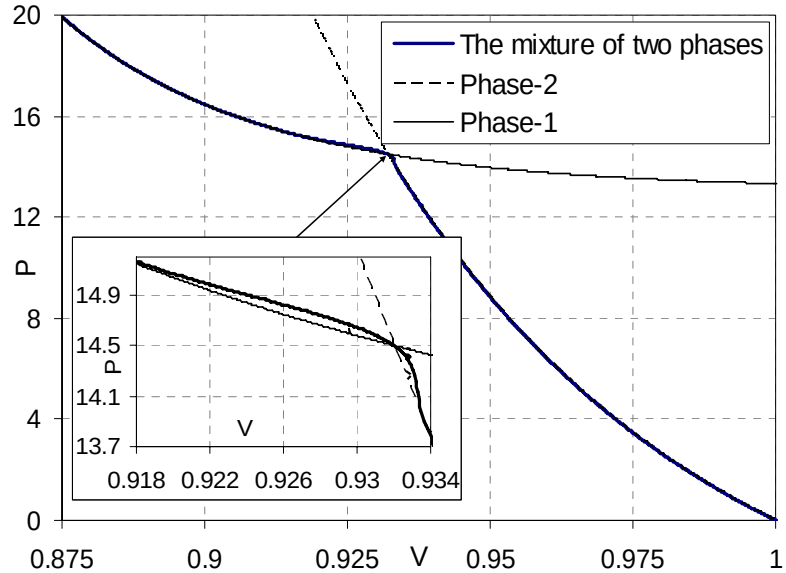

Figure 5 – The two-phase equation of state with intersection of phases for problem 3.

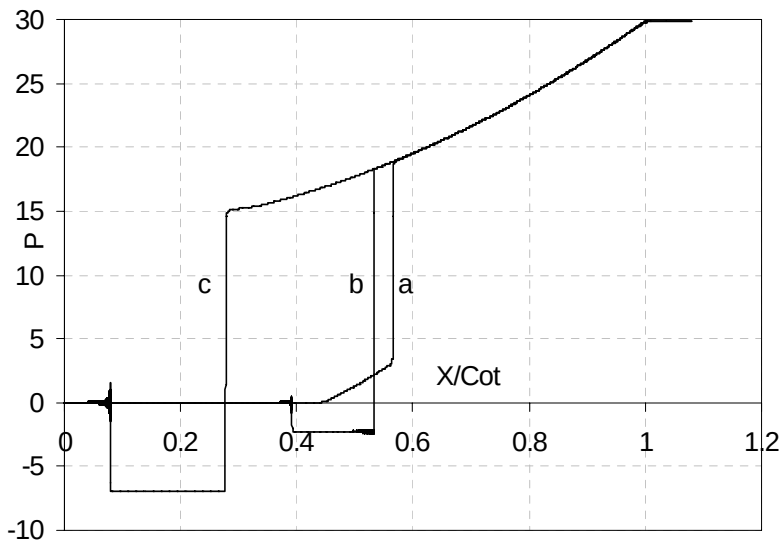

Figure 6 – The pressure profile in the centered rarefaction shock wave (problem 3) for the equation of state with intersection of phases. Computations with viscosity (a), dispersion (b), and interphase relaxation (c).

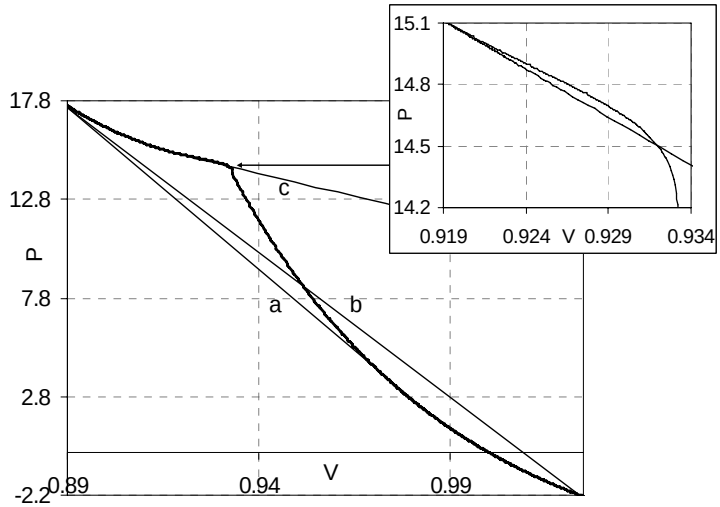

Figure 7 – The shock wave position in problem 3 computations with viscosity (a), dispersion (b, and interphase relaxation (c).

Conclusions made basing on the computation results

The resultant parameters of the rarefaction shock wave, in accordance with the results on the vanishing viscosity and vanishing dispersion methods, are determined by the relation between the approximation (or artificial) viscosity and approximation dispersion and, hence, depend on the difference scheme in use. The features of the vanishing dispersion method analytically proved in the paper and the features of the method of artificial interphase kinetics have been absolutely confirmed by 1D computations.

Features of main approximation error control in modern difference gas dynamics schemes are sufficient to simulate the required combinations of dissipation and dispersion processes inside the smeared shock wave front. Such control is possible in difference schemes of a rather high approximation order, not worse than the second order of approximation.

Conclusion

The results on the features of rarefaction shock waves in self-similar problems of the Riemann problem type known from literary sources are based on assumption that any discontinuous solutions to gas dynamics equations should be obtained from smooth solutions to gas dynamics equations with viscosity by transition to an infinitely small coefficient of viscosity , i.e. using the method of vanishing viscosity.

The efforts described in the paper resulted in the analytically obtained features of rarefaction shock waves for non-convex barotropic equations of state (when pressure depends on the specific volume only) for the two methods of regularization of gas dynamic equations of state – using the method of artificial normal dispersion and the method of artificial interphase kinetics.

It has been proved that the rarefaction shock waves obtained using the method of vanishing normal dispersion always differ from the rarefaction shock waves obtained using the method of vanishing viscosity. Even the velocities of such rarefaction shock waves obtained in self-similar Riemann problems are different.

The paper describes the method of smearing rarefaction shock waves in which artificial interphase kinetics is used instead of viscosity and dispersion, in assumption that a material consists

of two mixed phases with convex equation of state of each of the pure phases. It has been proved that if the pure phase EOSes don't intersect, the rarefaction shock waves obtained using the method of artificial interphase kinetics coincide with those obtained using the method of vanishing viscosity.

If the pure phase EOSes intersect, the rarefaction shock waves obtained with the method of artificial interphase kinetics significantly differ from those obtained with the method of vanishing viscosity.

The following should be emphasized here. As the obtained results show, various methods of extrapolation of the pure phase EOSes to the vicinity of phase transition and behind it (both with intersection of adiabatic curves of pure phases and without it) lead to absolutely different parameters of rarefaction shock waves with regard to the phase transition kinetics. Hence, such difference depending on a chosen model should be taken into account, when treating the experimentally obtained data, in particular, if there is an insufficient knowledge of the equations of state of pure phases in the phase transition area and if there some uncertainties of the phase transition kinetics parameters.

The results obtained in the paper prove that parameters of rarefaction shock waves depend on the processes inside the smeared rarefaction shock wave front. The results allow a unequivocal conclusion that the problem of choosing a single and "correct" rarefaction shock wave during simulation (including numerical simulation) of gas dynamic processes should be solved at the level of selecting physical models and depends on the major physical processes to be ignored, when writing the ideal gas dynamics equations. This conclusion is not new, in general. The review paper by Kulikovsky (1988) gives many examples of such a kind.

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