

Application of the Ceptre Code to Cable SGEMP Problems

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A computational capability has been developed for cable System Generated Electromagnetic Pulse (SGEMP) simulations. Specifically, CEPTRE (Coupled Electron-Photon Transport for Radiation Effects) is used to perform the radiation transport and provide charge and energy deposition profiles along with the net electron current off the conductors to the electrical response simulation code CABANA (CABLe ANALysis). An overview of CEPTRE is presented. An introduction to cable SGEMP analysis is given. The very nature of cable SGEMP calculations results in numerical difficulties for the unique solution strategy employed by CEPTRE and speed-up strategies have been developed. These strategies are discussed and demonstrated here for a specific cable SGEMP application.

Introduction

Electronic systems are designed for a wide variety of applications and environments, including in some cases radiation environments. Exposure to x-ray sources will cause photo/Compton-emission of electrons from material surfaces and the deposition of charge and energy within system materials. These currents and charges may result in the generation of electromagnetic fields producing electrical currents on cable shields and on the conductors within cables, a phenomenon referred to as cable SGEMP. A computational capability has been developed for a cable SGEMP simulation. Specifically, CEPTRE is used to perform the radiation transport and provide charge and energy deposition profiles along with the net electron current off of the conductors to the electrical response simulation code, CABANA.

CEPTRE, Coupled Electron-Photon Transport for Radiation Effects, is a time-independent, multi-group, deterministic coupled electron-photon transport code that solves a second-order formulation of the Boltzmann transport equation on unstructured finite element meshes. CEPTRE is a massively parallel, 2-D and 3-D, C++ object-oriented code employing spatial domain decomposition. It is integrated into the Nevada framework, formerly known as the Alegra framework (Boucheron et al., 1999). CEPTRE is designed to provide the radiation transport capability for subsequent mechanical and electrical analysis. The second-order formulation of the transport equation utilized by CEPTRE yields a symmetric positive definite (SPD) matrix, which allows the code to employ a unique solution strategy that simultaneously solves in the space and direction variables. The unique solution strategy employed by CEPTRE is characterized by fast

convergence rates for the electron transport and good parallel efficiency when compared to traditional source iteration strategies employed by most radiation transport codes.

Cable SGEMP simulations require accurate resolution of dose-enhancement and charge profiles near the conductor/dielectric interfaces. These requirements result in highly refined meshes at material interfaces with finite elements less than 1 micron in size and higher-order finite elements. However, these highly refined meshes are problematic for the photon transport yielding an ill-conditioned matrix system and very slow convergence in the photon energy groups. The CEPTRE code team is researching several techniques to speed-up the photon transport solution of cable SGEMP problems. These techniques include solution options using different preconditioners to speed-up convergence, solving the photon transport on a linear subset of the quadratic finite element mesh used for the electron transport, and employing lower-order angular quadrature for the photon transport.

In this paper, we provide an overview of CEPTRE including the second-order formulations of the Boltzmann transport equation it solves. A discussion of the unique solution strategy is presented in contrast to more traditional solution strategies employed by transport codes using the first-order formulation of the transport equation. We introduce cable SGEMP simulations, focusing on 2-D applications. A brief description of the CABANA code is offered. The nature of cable SGEMP analysis leads directly to numerical difficulties in the unique solution strategy employed by CEPTRE for photon transport. Strategies that the CEPTRE team has developed to counter these numerical difficulties are detailed and applied to a cable SGEMP simulation on the RG402 coaxial cable.

Overview of CEPTRE

CEPTRE is a time-independent, multidimensional, deterministic coupled electron-photon transport code developed under the ASCI program within the Nevada framework. The major features of CEPTRE are:

- multigroup energy discretization,
- discrete ordinates (S_N) angular discretization,
- arbitrary order of anisotropic scattering using a Legendre expansion of scattering cross-sections,
- unstructured-mesh using Galerkin finite elements,
- second-order forms of the Boltzmann transport equation,
- parallel implementation with spatial domain decomposition.

CEPTRE is an object-oriented program written in C++ and uses the parallel Krylov solver library Aztec (Tuminaro et al., 1999).

The Boltzmann transport equation solved by CEPTRE describes the particle distribution in the phase space. This particle distribution is referred to as the angular flux, $\Psi(\vec{r}, E, \hat{\Omega})$, which represents the flux of particles at the position \vec{r} , with energy E and traveling in the direction $\hat{\Omega}$. The discretization of the phase space is accomplished in CEPTRE through the multigroup approximation in energy along with the Legendre

expansion of scattering cross-sections, the discrete ordinates approximation in direction and finite elements in space.

The angular flux is the fundamental quantity that CEPTRE determines. The physics of particle-media interactions are properly characterized by cross sections generated using CEPXS, a cross-section generation code developed specifically for coupled electron-photon transport (Lorence et al., 1989). CEPXS generates multigroup-Legendre cross-sections; that is, directional dependency is approximated by a Legendre polynomial and the energy dependence is accounted for by approximating the energy over discrete groups (Lorence et al., 1989). Charge and energy deposition profiles induced by a radiation source on a given geometry are obtained by combining the scalar flux with the energy and charge deposition cross-sections, respectively.

The streaming-term, $\hat{\Omega} \cdot \nabla \Psi(\vec{r}, E, \hat{\Omega})$, in the traditional first-order equation is non-symmetric and non-positive-definite; therefore, applying FEM directly to this formulation results in a linear system that is non-symmetric and non-positive-definite. Historically, developing an efficient parallel code that solves the first-order Boltzmann equation is problematic. To avoid these inherent difficulties, CEPTRE employs two second-order formulations of the Boltzmann Transport equation: the Self-Adjoint Angular Flux (SAAF) equation and the Even-Odd Parity Formulation (EOPF). Applying finite element methods (FEM) to the second-order formulations of the transport equation yields linear systems that are SPD. SPD systems can be solved using robust, efficient numerical techniques such as conjugate-gradients (CG). Furthermore, parallel linear-equation solvers, such as the Aztec package (Tuminaro et al., 1999), can be directly applied to efficiently solving the linear, SPD system on massively parallel computers.

First-order Boltzmann transport equation

The angular flux can be determined by solving the time-independent, linear Boltzmann transport equation (Lewis and Miller, 1993) written as

$$\hat{\Omega} \cdot \nabla \Psi(\vec{r}, E, \hat{\Omega}) + \sigma_t(\vec{r}, E) \Psi(\vec{r}, E, \hat{\Omega}) = \int dE' \int d\Omega' \sigma_s(\vec{r}, E' \rightarrow E, \hat{\Omega}' \cdot \hat{\Omega}) \Psi(\vec{r}, E', \hat{\Omega}') + Q(\vec{r}, E, \hat{\Omega}) \quad (1)$$

where $\Psi(\vec{r}, E, \hat{\Omega})$ is the angular flux, $\sigma_t(\vec{r}, E)$ is the total cross-section,

$\sigma_s(\vec{r}, E, \hat{\Omega}' \cdot \hat{\Omega})$ is the differential scattering cross-section, which is the probability of scattering between different energies and directions, and $Q(\vec{r}, E, \hat{\Omega})$ is a specified external source.

Equation (1) is the first-order form of the Boltzmann transport equation, which is a mathematical statement of particle balance over a differential volume in the phase space of $(\vec{r}, E, \hat{\Omega})$. The first term on the left of Eq. (1), $\hat{\Omega} \cdot \nabla \Psi(\vec{r}, E, \hat{\Omega})$, is the streaming term which accounts for particle transport between interactions. The rate of removal is represented by the second term, $\sigma_t(\vec{r}, E) \Psi(\vec{r}, E, \hat{\Omega})$, which accounts for particle loss due to all interactions with matter. The total cross section is the probability per unit path length of a particle undergoing an interaction of any kind (e.g. absorption, scattering,

etc.). The integral term on the right-hand side of Eq. (1) accounts for scattering interactions which result in direction and/or energy changes. The scattering source effectively couples together all directions and energies. The presence of an external source is represented by the final term on the right-hand side, $Q(\vec{r}, E, \hat{\Omega})$.

Equation (1) is suitable for modeling neutral particle transport; however, it is not very efficient for modeling charged particle transport because of the extremely forward peaked scattering cross-sections that characterize these particles. This form of the transport equation can be used to efficiently model charged particle transport by employing Goudsmit-Saunderson modified electron cross-sections (Drumm, 1997). The Goudsmit-Saunderson theory allows charged particles to be modeled without an explicit continuous slowing down term (Morel, 1981) for problems of interest in cable SGEMP simulations. CEPTRE utilizes these Goudsmit-Saunderson modified electron cross-sections to effectively model electron transport in a neutral particle code.

Second-order formulations of the Boltzmann transport equation

CEPTRE uses two second-order formulations of the transport equation: (1) the SAAF equation and (2) the EOPF equation. CEPTRE employs the SAAF equation to perform first collision source (FCS) calculations which allow normally incident plane-wave photon sources to be adequately represented by standard quadrature sets. The FCS calculations yield an uncollided flux (Lewis and Miller, 1993) which is used to determine a distributed source over the quadrature angles for the full transport solve performed using the EOPF equation.

The SAAF equation is derived by appropriate manipulation of Eq. (1) (Morel and McGhee, 1999). The SAAF equation is a differential-integral equation for the angular flux that contains a second-order differential operator in space:

$$-\hat{\Omega} \cdot \nabla \left[\Re^{-1} \hat{\Omega} \cdot \nabla \Psi(\vec{r}, \hat{\Omega}) \right] + \Re \Psi(\vec{r}, \hat{\Omega}) = Q(\vec{r}, \hat{\Omega}) - \hat{\Omega} \cdot \nabla \left[\Re^{-1} Q(\vec{r}, \hat{\Omega}) \right] \quad (2)$$

where the removal operator \Re is defined as

$$\Re \Psi(\vec{r}, \hat{\Omega}) = \sigma_t(\vec{r}) \Psi(\vec{r}, \hat{\Omega}) - \int d\hat{\Omega}' \sigma_s(\vec{r}, \hat{\Omega}' \cdot \hat{\Omega}) \Psi(\vec{r}, \hat{\Omega}'). \quad (3)$$

The EOPF equations are derived in a manner analogous to the SAAF equations (Ackroyd, 1997) and have the same structural form:

$$-\hat{\Omega} \cdot \nabla \left[\Re_o^{-1} \hat{\Omega} \cdot \nabla \Psi^E(\vec{r}, \hat{\Omega}) \right] + \Re_E \Psi^E(\vec{r}, \hat{\Omega}) = Q^E(\vec{r}, \hat{\Omega}) - \hat{\Omega} \cdot \nabla \left[\Re_o^{-1} Q^E(\vec{r}, \hat{\Omega}) \right] \quad (4)$$

$$-\hat{\Omega} \cdot \nabla \left[\Re_e^{-1} \hat{\Omega} \cdot \nabla \Psi^O(\vec{r}, \hat{\Omega}) \right] + \Re_o \Psi^O(\vec{r}, \hat{\Omega}) = Q^O(\vec{r}, \hat{\Omega}) - \hat{\Omega} \cdot \nabla \left[\Re_e^{-1} Q^O(\vec{r}, \hat{\Omega}) \right] \quad (5)$$

where the even- and odd-parity fluxes are defined as

$$\Psi^E(\vec{r}, \hat{\Omega}) = \frac{1}{2} \left[\Psi(\vec{r}, \hat{\Omega}) + \Psi(\vec{r}, -\hat{\Omega}) \right] \quad (6)$$

$$\Psi^O(\vec{r}, \hat{\Omega}) = \frac{1}{2} \left[\Psi(\vec{r}, \hat{\Omega}) - \Psi(\vec{r}, -\hat{\Omega}) \right] \quad (7)$$

The even- and odd-parity fluxes are symmetric and asymmetric in $\hat{\Omega}$, respectively, allowing the angular flux to be modeled over half the angular domain. Thus, when we apply the discrete ordinates method to Eq. (4) and (5) the number of discrete directions is *half* that for the angular flux. For the EOPF formulation an S_8 problem means the angular flux is solved at 20 distinct directions as opposed to 40 directions for the SAAF equation or the traditional first-order equation.

We note that the operators in both the SAAF and EOPF equations are self-adjoint; discretizations of these equations may yield matrices that are SPD. We also observe that the scattering term appears in the left side of the equations; there are no unknown variables on the right side.

CEPTRE solution strategy

As observed above, CEPTRE solves transport equations in which the within-group scattering term is included with the streaming operator and removal terms on the left side of the equation. By incorporating this term with the removal and streaming operators we may avoid traditional source iteration techniques, which are notoriously slow for electron transport. Figure 1 illustrates the differences between the novel solution strategy for CEPTRE and the traditional source iteration approach. Utilizing a second-order formulation, which gives an SPD coefficient matrix when FEM are applied, allows CEPTRE to employ a unique solution strategy in which the space-angle dependence is solved simultaneously by using the conjugate gradient method (Drumm and Lorenz, 1999). Furthermore, the Aztec (Tuminaro et al., 1999) solver package can be directly applied to solve the linear system efficiently on massively parallel computers.

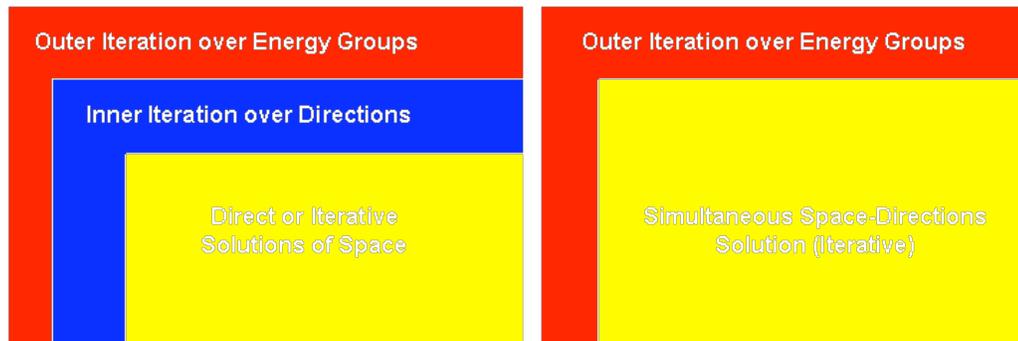


Figure 1. Conventional Source Iteration Compared to CEPTRE Approach.

In CEPTRE, the linear system is solved by building a matrix which includes both the space and angular dependence. The linear system is characterized by a sparse block matrix that is SPD. The number of block rows is equal to the number of nodes (N_{nodes}) in the FE mesh. Each of the block sub-matrices are $N \times N$ square, where N is the number of directions, and each block is full due to the coupling from scattering. The block coefficient matrix is tailor made for the distributed variable block row (VBR) data storage format available in Aztec (Tuminaro et al., 1999).

It is obvious the modeling of realistic geometries will result in very large matrix systems which require a large amount of storage. The storage requirements for CEPTRE grow in proportion to $N^2 \times N_{nodes}$ which means parallel efficiency is paramount. The run-time for CEPTRE also increases in proportion to $N^2 \times N_{nodes}^{1.5}$.

CEPTRE employs a preconditioned conjugate-gradient solver with domain decomposition to solve the coupled space-angle linear system in parallel. This approach works extremely well for electron transport. However, for photons the CG convergence can be unacceptably slow, consuming as much as 90% of the runtime. The difficulty CEPTRE encounters in solving photon transport efficiently is directly related to the physics that characterizes electron and photon transport. There is an enormous difference between the track lengths of electrons and photons. Photon transport is characterized by a quantity known as the mean free path (MFP), which is the reciprocal of the total cross-section, and represents the average distance between collisions. Electron transport is characterized by a quantity called the range, which is the average distance an electron travels before being absorbed. For the same energy, the photon MFP and the electron range are very different. Table 1 demonstrates the difference in these quantities for both a high-Z material (Ag) and low-Z dielectric material (PTFE, a carbon-fluorine composite) at energies typically encountered in cable SGEMP analysis.

Table 1. Comparison of Photon MFP and Electron Range at 100 and 30 keV

Energy (keV)	Photon MFP (μm)		Electron Range (μm)	
	PTFE	Ag	PTFE	Ag
100	29,797	633	78.9	23.8
30	10,479	26.6	9.8	3.2

To accurately model the electron transport at material interfaces, particularly between high-Z and low-Z materials, highly refined FE cells are required. When the FE mesh cells are highly refined and the thickness of the cells with respect to the MFP approaches zero the matrix becomes ill conditioned resulting in very slow convergence in the photon groups. These extremely small cells are transparent to the photons. As indicated in Table 1, for a typical SGEMP application, electron total cross sections may be three or four orders of magnitude larger than those for photons.

Next, we introduce cable SGEMP analysis, along with the purpose of performing these simulations. The computational requirements of cable SGEMP analysis directly lead to situations in which the FE mesh is poorly suited for photon transport and reaching convergence in the photon groups routinely consumes most of the runtime.

Cable SGEMP Analysis

Incident x-ray sources on electronic systems can cause photo/Compton-emission electrons from material surfaces and energy and charge deposition within its materials, potentially damaging or disrupting the system. In this article we are concerned with a phenomenon called cable SGEMP, which is observed when the interaction of x-rays within or near cables results in the emission of electrons and deposition of charges, generating electromagnetic (EM) fields on cable shields and conductors within cables.

The incident photons create EM fields in cables by two mechanisms: 1) electron displacement within the dielectric, subsequently inducing current within nearby

conductor regions, and 2) knock-on charge generated by photon-induced electrons created near the conductor/dielectric interface and transported into the dielectric. SGEMP is observed as the EM fields drive charge off/on conductors and deposit charge and induce conductivity in dielectric materials within the outer cable shield which results in potentially large transient currents being induced on internal cable conductors. The radiation effects community also refers to Cable SGEMP as Direct Charge Injection (Turner and Scrivner, 2001). In cable SGEMP simulations, we are interested in the direct-drive component of SGEMP, which is the resulting Load Charge as depicted in Figure 2.



Figure 2. Direct-Drive Component of Cable SGEMP Simulations.

A robust cable SGEMP computational simulation capability has been created at Sandia National Laboratories. This computational capability consists of the loosely coupled codes CEPTRE and CABANA. Specifically, CEPTRE performs the radiation transport and CABANA determines the electrical response.

CABANA

CABANA was developed at Sandia explicitly to address Cable SGEMP phenomena (Turner and Scrivner, 2001). CABANA solves the electrical portion of a cable SGEMP simulation. CEPTRE provides specific radiation transport results to CABANA consisting of energy deposition, current and charge profiles induced by radiation. The electrical response of the cable to an arbitrary load is calculated by CABANA which solves the quasi-static Maxwell equations using FEM techniques (Turner and Scrivner, 2001).

CEPTRE provides CABANA with the charge and energy deposited within the dielectric layers of a cable, as well as the net current from each of the cable's internal conductors. These radiation transport results represent the total integrated values over a specified radiation pulse. CABANA applies the time dependence to the simulation by assuring, for example, that the total charge in a given element is applied over the specified pulse shape (Turner, 2003). Radiation-induced conductivity (RIC) in the dielectric layers can also be calculated by CABANA (Turner, 2003).

CABANA computes the knock-on/off currents from the internal conductors and the rate of charge and energy deposition within the dielectric materials as a function of time (Turner and Scrivner, 2001). The SGEMP electrical response is quantified by the final load charge present at the end of the radiation pulse. It should be noted that CABANA

requires FE that are 2nd order, utilizing quadratic shape functions to obtain sufficient accuracy in the computation of electric field (Turner, 2003). A more detailed description of CABANA is beyond the scope of this paper and the reader is referred to the CABANA users guide for more details (Turner, 2003).

Accuracy requirements of cable SGEMP simulations

Cable SGEMP simulations require accurate resolution of dose-enhancement and charge profiles near the conductor/dielectric interfaces. Conductors are high-Z materials compared to the dielectric materials which are low-Z. Sharp gradients occur in charge deposition distributions at these conductor-dielectric material interfaces as a direct result of the significant difference between the photon MFP and the electron ranges as they move from high-Z materials to low-Z materials (see Table 1). The short range of electrons requires highly refined FE cells in order to accurately model their transport.

Figure 3 illustrates typical refinement at material interfaces for the RG402 coaxial cable. The FE mesh depicted in Figure 3 is converged for a particular incident photon spectrum with an average energy of 56 keV and the finest layer of cells is $\approx 0.18 \mu\text{m}$ thick. These highly refined meshes at material interfaces, typically using FE cells less than $1 \mu\text{m}$ in size, pose a problem for the photon transport yielding an ill-conditioned matrix system and very slow convergence in the photon energy groups.

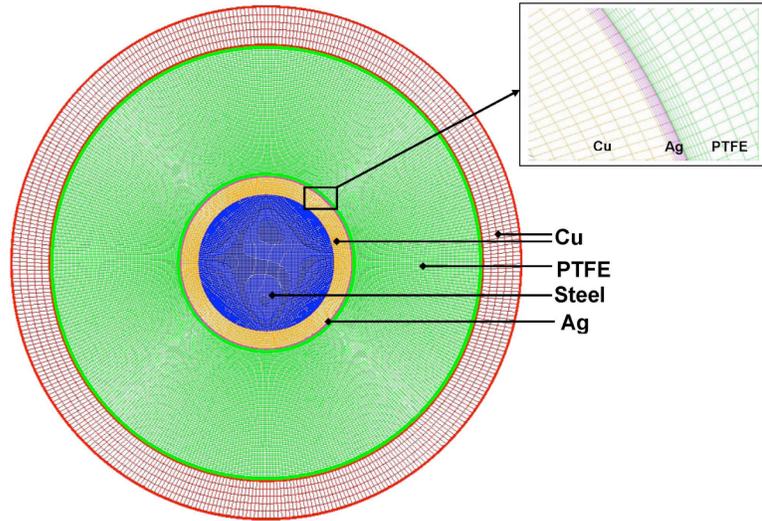


Figure 3. Example of Mesh Refinement at Dielectric-Conductor Interface for RG402 Coaxial Cable.

CEPTRE also utilizes higher-order finite elements, using quadratic basis functions, to accurately model the electron transport at these material interfaces. These higher-order FE contain more nodes which translates into higher computational costs simply because

the transport equation is being solved at more nodes (recall the run-time for CEPTRE increases in proportion to $N_{nodes}^{1.5}$).

Figure 4 illustrates the significant cost of the CG photon solve on the FE mesh optimized for electron transport shown in Figure 3. For the CEPTRE calculation represented in Figure 4, 95% of the run-time was spent on the photon energy groups. The CEPTRE team is exploring several techniques to speed-up this runtime to allow analysts to perform bounding calculations in a timelier manner. In the next section we will discuss these strategies.

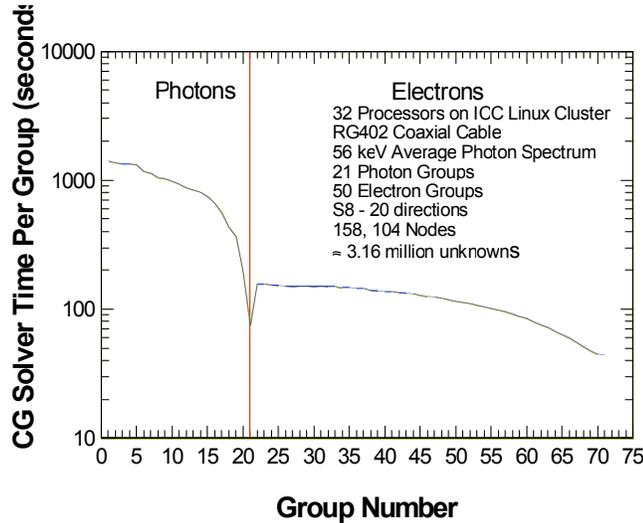


Figure 4. CG Solver Time per Energy Group for a RG402 Coaxial Cable.

CEPTRE Speed-Up Strategies

The CEPTRE team is researching several techniques to speed-up the photon transport solution of cable SGEMP problems. These techniques include solution options using different preconditioners to speed-up convergence, solving the photon transport on a linear subset of the quadratic finite element mesh used for the electron transport, and employing lower-order angular quadrature for the photon transport. The primary preconditioning technique the CEPTRE team is investigating is using a block diagonal preconditioner to accelerate convergence of the photon energy groups. The preconditioner matrix is derived by applying Galerkin finite elements to the transport equation without scattering and is much easier to solve than the block system arising from the original transport equation. There is concern that solver techniques employed by CEPTRE to speed-up the run-time might affect the overall accuracy of the cable SGEMP simulation and the calculated electrical response of the cable. For each of these solution strategies, we want to assess how much speed-up is observed in the photon solve and the effects on the electrical response of the cable.

Solve photons on linear finite element mesh

The higher-order quadratic finite elements are necessary to accurately model electron transport at material interfaces and capture the sharp gradients in charge deposition

profiles. Linear finite elements are sufficient to model photon transport accurately. CEPTRE has the capability to solve the photons on a linear subset of the quadratic FE mesh optimized for electrons. Essentially CEPTRE solves the photon transport on a linear version of the quadratic mesh provided for a problem. A simple linear mapping is used to map the photon solution to the quadratic mesh for the electron transport. The runtime savings in this strategy is simply due to the reduction in nodes, and thus the smaller linear system that must be solved for the photon groups.

Lower-order angular quadrature for photon transport

At the energies of interest for cable SGEMP simulations photon scattering interactions are more isotropic and a low-order P_L approximation is adequate to model the scattering process. However, electrons are characterized by extremely forward-peaked scattering and require higher order Legendre expansions to adequately capture the degree of anisotropy. The order of the P_L approximation directly affects the S_N order an analyst uses. The higher the Legendre order the more angles needed in order to capture the degree of anisotropy accurately. For most cable SGEMP problems, electron transport can be modeled accurately with S8/P5 approximations. However, photon transport can be modeled with S6/P3 and even S4/P3 approximations. Since the blocks in the linear matrix system are $N \times N$, where N is the number of directions, and the runtime is proportional to N^2 reducing the quadrature order for the photon energy groups will reduce the runtime.

Block diagonal preconditioner for CG

Outside of the Nevada framework, investigations have been done using the uncollided-flux as a preconditioner. The uncollided-flux operator contains only the streaming and removal terms of the transport operator. Without the scattering terms, it is simpler to solve because the directions are all decoupled and may be solved independently (Drumm and Fan, 2003). Thus, the work is shifted to solving a space-only matrix for each discrete direction. The preconditioner solve is also performed with CG, and because the preconditioner may also be ill-conditioned, convergence of the inner CG iterations may be slow. However, because the runtime of the preconditioner solution scales linearly with the number of directions, and the full space-direction solution scales with the square of the number of directions, the preconditioned solve generally wins out, especially for higher S_N orders (Drumm and Fan, 2003).

The preconditioned CG algorithm replaces the linear solve of $Ax = b$ with

$$M^{-1}Ax = M^{-1}b \quad (8)$$

where M is an SPD preconditioner. The preconditioned linear system must be better conditioned than the original linear system. Also an effective preconditioner must be easier to solve than the original system which is true of the uncollided-flux preconditioner. The uncollided flux operator is derived from the uncollided flux equation:

$$-\hat{\Omega} \cdot \nabla \left[\frac{1}{\sigma_t(\vec{r})} \hat{\Omega} \cdot \nabla \Psi(\vec{r}, \hat{\Omega}) \right] + \sigma_t(\vec{r}) \Psi(\vec{r}, \hat{\Omega}) = Q(\vec{r}, \hat{\Omega}) - \hat{\Omega} \cdot \nabla \left[\frac{1}{\sigma_t(\vec{r})} Q(\vec{r}, \hat{\Omega}) \right] \quad (9)$$

There is no coupling between directions because scattering is not accounted for in Eq. (9). The elimination of the scattering source greatly simplifies the solution. The resulting operator, M , is a sparse-block matrix that is SPD and resembles the original matrix system except for the fact that the directions are decoupled.

RG402 Cable SGEMP Simulation

A 1-D cross section of the RG402 coaxial cable is presented in Figure 5 along with the physical dimensions of the cable. A radial cross-section of this cable is depicted in Figure 3, presented earlier, which shows the finite element mesh of quadratic, quadrilateral elements (QUAD8) used for this simulation. The refined coaxial mesh used for this simulation is made of 52,569 QUAD8 finite elements and 158,104 nodes. The finest layer of elements at the conductor/dielectric interface (Ag/PTFE) is $\approx 0.18 \mu\text{m}$ thick. The FE mesh used has been shown to be converged for CEPTRE calculations using the incident photon spectrum chosen with an average energy of 56 keV and endpoint energy of 200 keV.

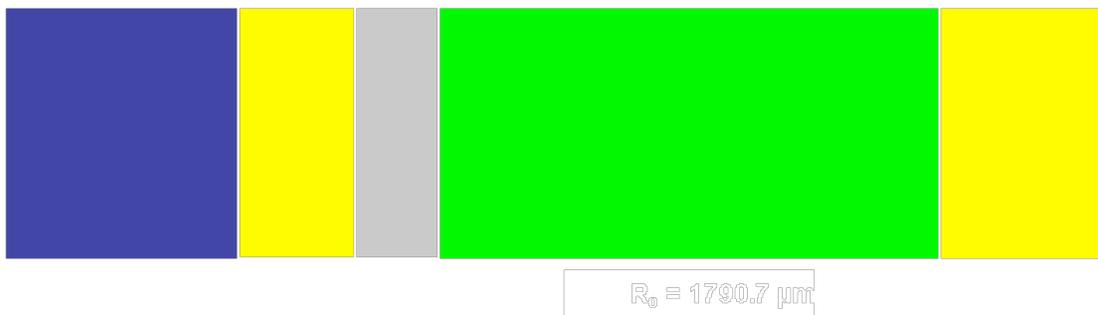


Figure 5. 1-D Cross-Section of RG402 Coaxial Cable.

We will perform the same cable SGEMP simulation on this RG402 cable using different speed-up strategies and evaluate the electrical response of the cable with CABANA. The idea is to see how much the electrical response changes as we employ different speed-up strategies to the radiation transport calculation. The hope is that an analyst will be able to perform several calculations, bounding the electrical response, in a matter of days as opposed to the weeks or months that it takes without these speed-up strategies. It should be noted that it is not unusual to run several CEPTRE calculations on different FE meshes to ensure that the energy group discretization and mesh is converged. For each simulation performed we will present the CEPTRE result as the knock-off charge at the conductor/dielectric interface (Ag/PTFE interface), and the CABANA result is the final load charge (SGEMP) given in nC/cm.

RG402 baseline simulation

The baseline RG402 simulation is performed with an S8/P5 approximation on a highly refined mesh of QUAD8 finite elements. The surface source is normally incident on the cable with an average energy of 56 keV; the incident photon spectrum ranges from 200 keV to 10 keV. The calculation uses 32 processors on Sandia’s Institutional Computational Cluster (ICC) which is a Linux cluster made up of 236 dual 3.06 GHz Xenon processors with 2 GB RAM. The CEPTRE calculation took 69.07 hours to complete of which 65.75 hours, 95% of the total runtime, were spent in the CG solve of the photon energy groups. The knock-off charge in electrons/cm and the final load charge or SGEMP in nC/cm are presented in the first row of Table 2. These are the baseline CEPTRE and CABANA results against which we will compare the other simulations.

Table 2. Comparison of Speed-up Strategies on CEPTRE and CABANA Results and Photon Solve Times

Photon SN / PL	Q knock-off (electrons/cm)	SGEMP (nC/cm)	Photon Solve Time (hrs)	Photon Solve FEM
S8 / P5	3.39e-04	6.24e-17	65.75	quadratic
S8 / P5	3.47e-04	8.14e-17	5.21	linear
S8 / P3	3.04e-04	8.25e-17	5.33	linear
S4 / P3	3.04e-04	8.07e-17	0.88	linear

Solving photons on linear finite elements

The first speed-up strategy is to simply solve the photons on linear, rather than quadratic, quadrilateral finite elements (QUAD4). This is a simple option turned on in CEPTRE by a keyword in the input file. The physics modeled are exactly the same as with the baseline calculation. This CEPTRE calculation finished in 8.40 hours which is 8.2 times faster than when both the photons and electrons are solved on QUAD8 finite elements. These results are presented in the second row of Table 2. The CEPTRE results are within 2.3% of the baseline calculation while the electrical response is within 30%. This shows how sensitive the electrical response is to the radiation transport result.

The sensitivity of the electrical response to the radiation transport results is due to the fact that the motion of the charge through the dielectric region is very small. A slight change in this minute charge motion translates into a larger, and at times significant, change in the response of the cable.

Solving photons with lower order Legendre approximation

The next speed-up strategy is to solve the photons with a lower order Legendre approximation, which translates into a lower S_N order. The CEPTRE and CABANA results are presented in the third and fourth rows of Table 2. When only the Legendre

order is reduced there is no additional savings in run-time, but when the S_N order is reduced along with the Legendre order from S8/P5 to S4/P3 an additional speed-up of about 6 is observed in the photon groups; the overall speedup from the baseline calculation is about 75. For the S4/P3 photon simulation the CEPTRE result is within 10.3% and the SGEMP response is still within 30%. For an analyst who wants to perform bounding calculations this is a very positive result allowing a reasonable response to be obtained when trying to converge on a mesh in CEPTRE in a few hours as opposed to days.

Using the UCF block preconditioner

Outside of the Nevada framework, Drumm and Fan (2003) have assessed the use of a UCF block preconditioner to speed-up the CG solve of the photon groups. The RG402 problem was run with S8/P3 and S16/P3. In the S8 simulation the CEPTRE calculation observed a speed-up of 4.3 using 80 ASCII Red processors when the UCF preconditioner was employed. Similarly, for the S16 case a speed-up of 8 was observed on 80 ASCII Red processors. The more taxing the problem is in the CG solve, the more effective the UCF preconditioner appears to be. The cable electrical response is not affected by using the UCF preconditioner alone as the same solution is reached by CEPTRE if it is truly converged. Work is underway to implement this preconditioner in the Nevada framework version of CEPTRE which requires modifications to third party libraries.

Conclusions

Performing accurate cable SGEMP simulations is being done using CEPTRE and CABANA. The differences in the photon MFP and electron ranges make determining accurate charge deposition profiles and dose enhancement at material interfaces between conductors and dielectrics very challenging. Highly refined FE meshes must be used at these material interfaces along with higher order quadratic finite elements which result in very slow convergence in the photon groups. CEPTRE can speed-up this solution with acceptable changes in the electrical response by solving the photons on linear FE and using lower order quadrature approximations. This speed-up allows analysts to perform several bounding calculations for a specific cable in a matter of hours or days as opposed to weeks. The UCF block preconditioner also yields impressive speed-up of the CEPTRE solution becoming even more effective as the CEPTRE problem becomes more and more difficult (Drumm and Fan, 2003). The CEPTRE team continues to evaluate these strategies for more cable SGEMP simulations. Currently, active work is being done to implement first-order S_N transport for photons in parallel. The solution of the first-order S_N transport equation using source iteration is fast for photon transport. The first-order S_N capability will also allow the photon and electron transport to be solved on the same FE mesh removing the need to map the photon solution to a quadratic mesh.

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