Numerical Simulation Methods Used at VNIIEF for Multidimensional Radiation and Particle Transport Problems

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The paper reviews the numerical simulation methods used at RFNC-VNIIEF for various categories of multidimensional transport problems (linear and nonlinear, time-dependent and time- independent, etc.). It briefly describes, in particular, some specific features of application of the methods, such as Monte Carlo method and the method of angular coefficients (view factors), and gives a detailed analysis of deterministic grid methods. Special emphasis is put on the issues concerning nonlinear multidimensional time-dependent linked problems, where many other physical processes are considered along with the transport process.

Introduction

The processes of particle transport and interaction with medium are among the most complex ones and numerical solution of such problems is of high labor intensiveness. The paper presents summaries of the numerical methods for transport problems developed at RFNC-VNIIEF. Various approaches to simulation of transport processes depending on the category of application problems and with regard to the requirements imposed (completeness of a physical model, accuracy and cost-efficiency of computations, etc.) are used at RFNC-VNIIEF. First of all, these are the approaches based on the Monte Carlo method, the method of angular coefficients (view factors) and grid approximations to the original transport equation using the finite difference and finite element methods (deterministic grid methods). The paper briefly describes some specific features of using the first two approaches. The deterministic grid methods basing on the transport equation approximation with the methods of finite differences and finite elements are described in more details, with especial attention being paid to the numerical schemes and algorithms developed at RFNC-VNIIEF and intended for multidimensional linked problems. Consideration of many other physical processes along with the transport process is necessary to solve such problems and this imposes a number of strict requirements on the methods and algorithms to be developed.

Monte Carlo Method for Numerical Solution of Transport Problems

A separate report will be devoted to the description of the specific features of Monte Carlo method application at our institute. So, I give here only a brief description of the work lines on transport process simulation using Monte Carlo method. This method is extensively used at our institute (Zhitnik et al., 1999), (Kochubey et al., 2000), (Donskoy et al., 1993) to solve problems belonging to the following categories:

1.Linear problems of particle transport (transport of neutrons and photons, computations of critical parameters K_{eff} and λ, computations of protection against gamma-neutron radiation, computations of nuclear radiation safety of containers for transportation and storage of spent nuclear fuel, etc.).

2.Transport of charged particles.

3.Solution of linked problems, where the transport process is considered along with many other physical processes.

We developed a number of codes to solve these problems using Monte Carlo method. The algorithms implemented in these codes provide a high enough accuracy of physical process descriptions and are highly efficient. To simulate trajectories, the method of maximum cross-sections is used, it provides actually the same speedup with the use of both spectral and multigroup constants. Thermal motion of medium nuclei is considered during simulation of particle trajectories on cold cross-sections of a substance (Ivanov et al., 2003). This allows elimination of computations of cross-sections at given temperatures. The method of catastrophic collisions with Fokker-Planck approximation applied to describe collisions with small transfers of energy and momentum (Donskoy et al., 1993) is used for simulation of electron trajectories. There has been developed the model that simulates generation of annihilation and bremsstrahlung photons and allows a rather accurate description of their distribution without simulation of trajectories of electrons and positrons.

The code has been parallelized using MPI library of interprocessor communications. The algorithms in use demonstrate high parallelization efficiency using a large enough number of processors.

Another area of Monte Carlo method application to solution of transport problems is the development of simulation algorithms using grid geometries. The developed algorithms allow simulation using arbitrary grid geometries and the simulation efficiency actually doesn't depend on the grid cell sizes.

Computations of Radiation Transport in Vacuum Using the Method of View factors

When solving engineering heat transfer problems and laser target problems, it is required to consider the radiation transport in air holes of designs. For this purpose, the integral equation of radiation transport through an optically transparent medium (Babayev et al., 1978), (Zel'dovich et al., 1966), (Siegel et al., 1975), (Dementiev et al., 1984), (Babayev et al., 1995), (Bazin et al., 1998), (Dementiev et al., 1983) should be solved in such regions:

$$J_\nu^-(P,t) = \int_{S(P)} J_\nu^+\left(Q, t - \frac{\rho_{PQ}}{c}\right) \frac{\cos(\vec{n}_p, \vec{PQ})\cos(n_Q, \vec{QP})}{\pi r^2(P,Q)} dS_Q, \tag{1}$$

where $J_\nu^+(P,t)$, $J_\nu^-(P,t)$ are single-direction frequency radiation fluxes. Further, let the optically transparent regions be called vacuum regions and the rest ones be diffusion regions, for the sake of simplicity. On the vacuum region/diffusion region interface the equation (1) is added by the balance equation (2):

$$J^-(A,t) - J^+(A,t) = q, \tag{2}$$

where q is the energy flow across unit area of the boundary surface.

To solve the problem, the grid partition is specified on the vacuum region boundary. The surface grid of boundary components for vacuum regions is irregular, with the boundary components being arbitrary plane polygons.

Solution to the integral equation in combination with the balance equation on the interface between the vacuum and diffusion regions is to be found using the method of view factors (zonal method). The integral radiation transport equation is approximated by the system of linear algebraic equations relatively averaged for N boundary components of radiation flows:

$$S_i J_i^-(t) = \sum_{j=1}^{N} a_{ij} J_j^+\left(t - \frac{\rho_{ij}}{c}\right). \tag{3}$$

The following boundary conditions close the system of equations:

$$J_i^-(t) - J_i^+(t) = q, \qquad i = 1,\ldots,N. \tag{4}$$

Factors a_{ij} in the system of equations are the so-called view factors:

$$a_{ij} = \frac{1}{S_i} \iint_{Si\,Sj} H_{ij} \frac{\mu_p \mu_q}{\pi \rho_{PQ}^2} ds_q ds_p \tag{5}$$

In these factors, function H_{ij} allows taking into account possible shading of the view region between surfaces S_i и S_j by other bodies. The two known methods – the half-sphere method and the half-cube method - have been implemented to determine the view regions in integrals of equation (5) and calculate the view factors. Consider two problems of X-ray radiation transport in a cylindrical laser target schematically shown in Fig.1; they can serve as an example of using the radiation transport computation technique.

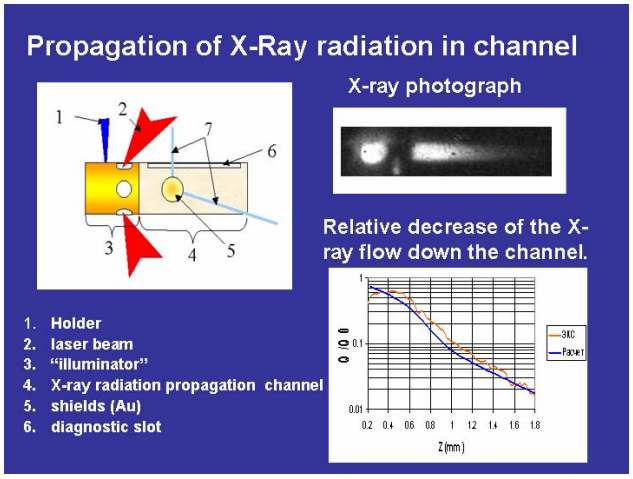


Figure 1. Propagation of X-ray radiation in channel

Numerical solution of multidimensional transport problems using the finite difference and finite element methods

Grid methods (Shagaliev et al., 1999) are extensively used to solve application problems belonging to various categories. The methods based on finite difference and finite element approximations to the multigroup transport equation should be mentioned first among such numerical methods. These methods are extensively used at VNIIEF to solve a wide range of application problems, in particular, for numerical solution of linear and nonlinear spectral transport problems in physics of high densities and energies. Such computations are carried out, as a rule, in multigroup approximation in 2D and 3D geometries along with simultaneous consideration of many other physical processes. With regard to these specific features, strict requirements of a high quality of grid methods to be developed and used are imposed.

The ideas underlying the numerical methods developed at VNIIEF to solve time-dependent transport problems are given below:

1. The time approximation to the time-dependent transport equation is constructed using the implicit two-point difference scheme.

2. The transport equation approximation in space is constructed using non-orthogonal spatial grids, namely:

- regular non-orthogonal grids of convex quadrangles;
- irregular non-orthogonal grids of arbitrarily shaped convex polygons.

The grids above were used to construct of a number of finite difference schemes: the extended template scheme (Pleteneva et al., 1989), (Moskvin et al., 2005); the scheme with introduction of closing relations based on moment equations; the scheme based on the use of adaptively refined grids in phase space (Shagaliev, 2004), (Shagaliev et al., 2004). For the transport equation discretization in angular variables, the schemes of the method of discrete ordinates are used. All the schemes mentioned above have common features consisting in that with the use of non-orthogonal grids they preserve important advantages of DS_n – schemes, such as the transport equation approximation within one cell of phase space and, hence, a possibility to resolve the system of grid equations using the scheme of sweep computations (point-to-point computations). At the same time, they

slightly differ from each other, in particular, in the accuracy of approximation using essentially non-orthogonal grids, support of the monotone behavior of the grid solution, implementation simplicity and in some other features.

3. The method of source iterations is used to find the numerical solution to the system of interrelated multigroup grid equations. The algorithms accelerating convergence of simple iterations are used to improve the efficiency of grid equation system computations. In order to accelerate convergence of simple iterations in case of linear time-independent problems, we have developed and successfully use the flow-consistent acceleration method, or FCA method (Evdokimov et al., 1994), (Evdokimov et al., 1996), (Moskvin et al., 1996) and for nonlinear time-dependent problems KM method and some modifications to this method have been developed (Fedotova et al., 1991).

4. The sweep method (point-to-point computations) and some of its modifications oriented to a multigroup case (Troshchieyev et al., 1976), (Fedotova et al., 1991) are used for numerically solving the system of grid equations with the known right-hand sides.

5. The effective combined parallelization algorithms oriented to a general case, when non-orthogonal spatial grids are used (Alekseyev et al., 2001), (Alekseyev et al., 1993), (Alekseyev et al, 1996), have been developed and used for numerical solution of 2D and 3D transport problems on multiprocessors.

The specific features of the computational algorithms and methods used to solve the transport equation with grid methods are described below in more details.

The transport equation approximation in time, space and angular variables. The extended-template schemes.

Such schemes are constructed using the values of the grid function in a quadrangular cell and on sides and at vertexes of this cell (Fig.2). The system of grid equations includes:

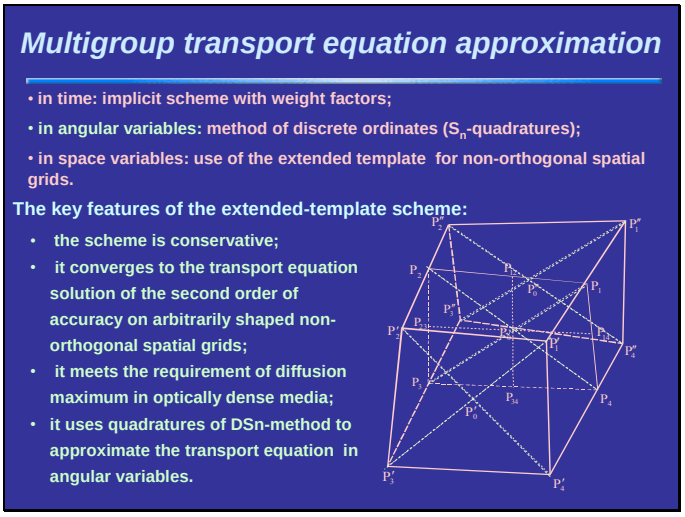


Figure 2. Multigroup transport equation approximation.

1. Grid equations of particle balance in grid cells.

2. Additional correlations in time and angular variable φ

3. Additional correlations in space variables that relate the desired function values in a cell, on sides and at vertexes of this cell to each other. The number of additional correlations in the scheme of interest depends on the number of illuminated sides of a quadrangular cell. Depending on the values of μ_m , $\varphi_{q,m}$, three variants of illumination of the spatial grid cell's sides are possible, they are shown in Fig.3.

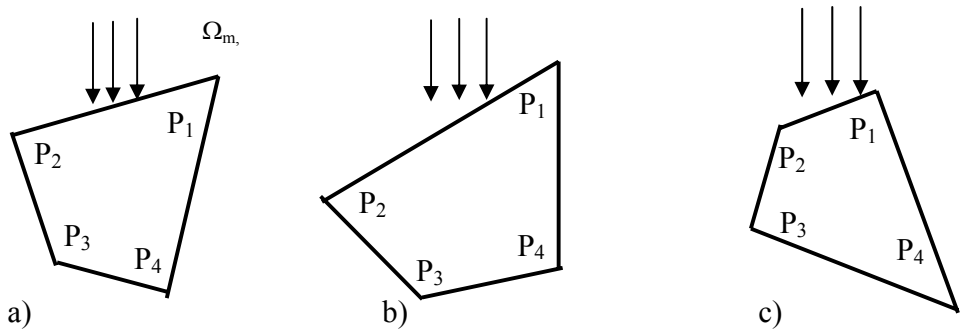


Figure 3. Possible variants of a cell side illumination**.**

In the two schemes with extended templates considered below, there are additional correlations corresponding to the illumination variants above and obtained using various methods. In scheme 1 (Pleteneva et al., 1989), additional correlations are used that are based on representation of the solution inside a cell in the form of a linear function. For the second scheme (scheme 2) (Moskvin et al., 2005) additional correlations are derived from an approximate representation of the solution to the transport equation along characteristics; this leads to the need in solving difference equations within one cell in two phases.

Below given are the computation results for one test problem of finding the eigen values of parameter λ (Pleteneva et al., 1989), (Moskvin et al., 2005). Numerical computations were carried out using rectangular grids and essentially non-orthogonal grids. The computation results are presented in Table 1. One can see that the calculated values of parameter λ obtained using the both schemes satisfy the formula corresponding to the second-order convergence, $\lambda = 0.1647 - \frac{A}{k^2}$, where A is a constant close to 1.

Table 1. The values of parameter λ in computations using scheme 1 and scheme 2 for various spatial grids.

Grid	Size of cells			
	h=0.1(см)	h/2	h/4	h/8
Values of λ for various spatial grids (scheme 1)				
Rectangular	0.1627515	0.1642543	0.1646291	0.1647228
Non-orthogonal	0.1618866	0.1639924	0.1644934	0.1646495
		ℓ=0.07cm	ℓ=0.035cm	ℓ=0.0175cm
Values of λ for various spatial grids (scheme2)				
Rectangular	0.162753	0.164256	0.164631	0.164724
Non-orthogonal	0.161205	0.164074	0.164678	0.16477
	ℓ=0.1cm	ℓ=0.07cm	ℓ=0.035cm	ℓ=0.0175cm

The scheme with introduction of closing relations based on the moment equations

When constructing this scheme, the solution inside a grid cell is represented in the form of a bilinear expansion in space variables and angular variable φ:

$$N = N_{Po} + \frac{\tau}{2} N_{\tau} + \frac{\eta}{2} N_{\eta} + \frac{\varphi'}{2} N_{\varphi'} + \frac{r\eta}{4} N_{r\eta} + \frac{r\varphi'}{4} N_{\tau\varphi'} + \frac{\eta\varphi'}{4} N_{\eta\varphi'} \ , \tag{6}$$

where $\varphi' = \dfrac{\varphi}{\Delta\varphi_{q-1/2}}, \eta, \tau$ are coordinates of a space point inside a grid cell during bilinear transformation of a quadrangle into a unit square. To find the unknown factors in the solution expansion, the balance equations (1)-(3) for particles in cells and the corresponding moment equations are used.

Some results of numerical studies using a linear problem of particle transport in a cylindrical layer $0 \le R \le 1,\ 0 \le Z \le 2$ are given below. The source and the equation coefficients are specified as $Q = 1,\ \alpha = 1, \beta = 1.$ A zero input flow is specified on the boundaries parallel to R-axis and the boundary condition “mirror reflection” is specified on the boundary parallel to Z-axis. Figure 4 shows the profile of density function for particles $n^{(0)}$ along the central column obtained using two schemes: the extended template scheme (scheme 1) and LMS scheme.

As one can see from this figure, LMS scheme doesn’t lead to underestimation of the solution on Z-axis and represents the solution constant to a high enough accuracy. At the same time, in computations using the extended template scheme there is some deviation from one-dimensional solution (the deviation is about 1% in computation 1), which is typical for DSn-type schemes.

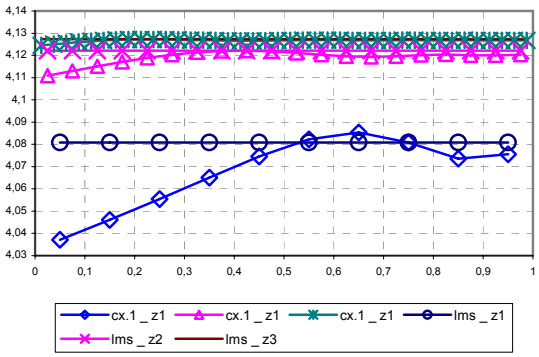


Figure 4. The solution along the central column.

The adaptive method of refined grids in phase space

The idea of the method is that in phase space, where the problem solution is to be found, some region (which is not simply connected, in general) is separated, where the original grid cells are refined to obtain cells of smaller sizes. (Shagaliev, 2004), (Shagaliev et al., 2004). Such refinement of the grid cells can be made in space variables, angular variables and energy variable.

For time-dependent problems, the region of the reference grid cell refinement is re-determined at time steps, with special algorithms being used for this purpose. When approximating the transport equation in refined cells of the adaptive grid, the key features of the scheme used for numerical solution of the transport equation using the reference grid are preserved, namely, the approximation order, conservativeness of scheme, a possibility to solve grid transport equations using the sweep method, and some others.

The example of one 2D time-dependent test problem is given below, computations were carried out using the adaptive method of grid refinement in space variables. The computational domain consists of two physical regions: region 1 is a dense casing $\{0 \le Z \le 5;\ 1 \le R \le 1{,}2\}$, region 2 is transparent $\{0 \le Z \le 5;\ 0 \le R \le 1\}$. Photon absorption is taken into account during computations, with the photon absorption cross-section being specified by the equation $\chi_a = A/T^3$, where $A=50.89$ in region 1 and A=0.1374 in region 2.

Figure 5. The system geometry for the 2D test problem.

The absorption cross-section was taken zero ($\chi_s = 0$) in computations. The initial temperature T at all points of the system was assumed to be 0,0001. An input isotropic

radiation flow corresponding to temperature $T = 1$ was specified for a part of the left end of the rectangular region. The equation describing the photon transport process and radiation interactions with medium was essentially two-dimensional in the problem under consideration.

To describe spatial grids used for computations, the following notations are used: $Nr(Pr)\times Nz(Pz)$, where Nr is the number of rows, Nz is the number of columns, Pr is the maximum adaptability level in rows (*MaxAdapt*), Pz is the maximum adaptability level in columns. The results of computations for the 2D test problem using adaptive grids of sizes 10(4)×50(4) and 10(8)×50(8) are shown in Fig.6, the numerical solutions obtained using some spatial grids without adaptive refinement are also shown, for comparison. As one can see from the plots presented, the solution obtained with the adaptive technique using the grid of size 10(8)×50(8) is close to the one obtained with the standard technique using the grid of size 40×200, with the adaptive computation requiring less time (by a factor of 2.8).

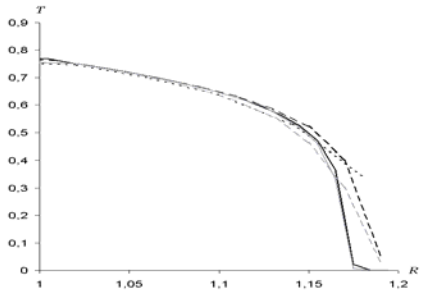


Figure 6. The material temperature profiles along line $Z = 2$ at time t=0,01 obtained for the 2D test problem using various reference grids: —— – basic grid 40×200; – – – – grid 20×100; ····· – grid 10×50; – – – – grid 10(4)×50(4); —— – grid 10(8)×50(8)

Algorithms accelerating convergence of iterative processes

To provide efficient computations of multigroup grid transport equations, it is important to use efficient convergence-accelerating algorithms of the traditionally used method of iterations in the right-hand side (source) of the transport equation.

For linear time-independent transport problems, we developed and successfully use the flow-consistent acceleration method (FCA method). Brief formulation of the idea of this method and some results of its numerical studies are given below.

FCA Method (the Flow-Consistent Acceleration Method).

In FCA method (Evdokimov et al., 1994), (Evdokimov et al., 1996) the desired functions are functions of a single-direction flow. The grid values of these functions are introduced on edges and mid lines of cells. The equations for the first moments and zero moment of function N containing the compensating sources to match the acceleration phase solutions and solve a simple iteration are introduced. Use of FCA method allows reduction in time for computations in optically dense systems.

Table 2 and 3 below give examples of FCA application to accelerate convergence of simple iterations (N/A method) in the test problem with pure scattering, which is solved within a cubic region, and in the 2D test problem of RBMK reactor.

Table 2. Region {*0<x<1.5, 0<y<1.5, 0<z<1.5*}

α	β	2D		3D	
		N/A	FCA	N/A	FCA
10.	9.	81	7	116	12
10.	9.9	331	7	300	13
10.	9.99	463	7	361	13
10.	10.	482	7	370	13

Table 3. Computation results for a channel of RBMK reactor.

Method	**Number of iterations**
Kellog method	229
Direct method	412
Method of iterations in source	555
Method of iterations in source + FCA	42
Method of iteration in source + FCA + + Chebyshev method	24

Convergence and efficiency of speedup methods are of critical importance, if one solves nonlinear time-dependent multigroup problems of X-ray radiation transport. For this category of problems we developed in the 80's and later improved the so-called KM method (Fedotova et al., 1991). The idea of this method is given below, along with the results of comparative computations for one time-dependent test problem.

KM method.

$$\frac{1}{v_i}\frac{\overset{s+\frac{1}{2}}{N_i^{n+1}}-N_i^n}{\Delta t}+L\,\overset{s+\frac{1}{2}}{N_i^{n+\gamma}}+\alpha_i\,\overset{s+\frac{1}{2}}{N_i^{n+\gamma}}=\frac{1}{2\pi}\sum_{j=1}^{NI}\beta_{ij}\,\overset{s}{n_j^{(0)^{n+\gamma}}},$$
$$N_i^{n+\gamma}=\gamma N_i^{n+1}+(1-\gamma)N_i^n,\ i=1,2,\ldots,NI \tag{7}$$

Phase I – predictor:

$$\frac{1}{v_i}\frac{\Delta\overset{s+1}{N_i^{n+1}}}{\Delta t}+L\,\Delta\overset{s+1}{N_i^{n+\gamma}}+\alpha_i\Delta\,\overset{s+1}{N_i^{n+\gamma}}-\sum_{j=1}^{NI}\beta_{ij}\Delta\,\overset{s+1}{N_j^{n+\gamma}}=\frac{1}{2\pi}\sum_{j=1}^{NI}\beta_{ij}\,\Delta\overset{s+\frac{1}{2}}{n_j^{(0)^{n+\gamma}}},$$
$$\Delta\overset{s+1}{N_i}=\overset{s+1}{N_i}-\overset{s+\frac{1}{2}}{N_i},\ \Delta\overset{s+1}{N_i^{n+\gamma}}=\gamma\,\Delta\overset{s+1}{N_i^{n+1}}, \tag{8}$$
$$\Delta\overset{s+\frac{1}{2}}{n_i^{(0)}}=\overset{s+\frac{1}{2}}{n_i^{(0)}}-\overset{s}{n_i^{(0)}}\ ,i=1,2,\ldots,NI$$

Phase II – corrector.

Table 4 gives the results of computations using KM method for one test problem.

Table 4. The results of computations with KM method for one test problem.

Step No.	$\Delta t = 0.1 \cdot 10^{-10}$ s			$\Delta t = 0.5 \cdot 10^{-10}$ s			$\Delta t = 0.1 \cdot 10^{-9}$ s		
	Pre-iteration	KM	KM3	Pre-iteration	KM	KM3	Pre-iteration	KM	KM3
1	19108	14	14	28696	26	26	26454	35	35
5	3332	24	20	6363	121	88	7960	238	174
10	1559	27	20	8477	101	75	9146	179	135
t(s)	*2880*	*61*	*47*	*7020*	*233*	*184*	*8580*	*435*	*347*

Parallelization algorithms

As it was already mentioned above, numerical simulation of many various categories of multidimensional time-dependent transport problems leads to a heavy computational burden. We developed effective methods of fine-grain parallelization oriented to a general case of using non-orthogonal spatial grids to the problems above in 2D and 3D space approximations on multiprocessor systems (Alekseyev et al., 2001) (Alekseyev et al., 1993) (Alekseyev et al., 1996). These parallelization algorithms are to be presented at the conference in a separate report. Here, I only demonstrate their efficiency by the example of one 3D test problem. The problem parameters are: 8 energy groups, 96 particle flight directions (S_8), 250000 three-dimensional spatial cells. The efficiency is estimated using the method of increase. The problem size remains unchanged on each processor. $Sp_N = \frac{T_1 * N}{T_N}$. The computations results are shown in Fig.7.

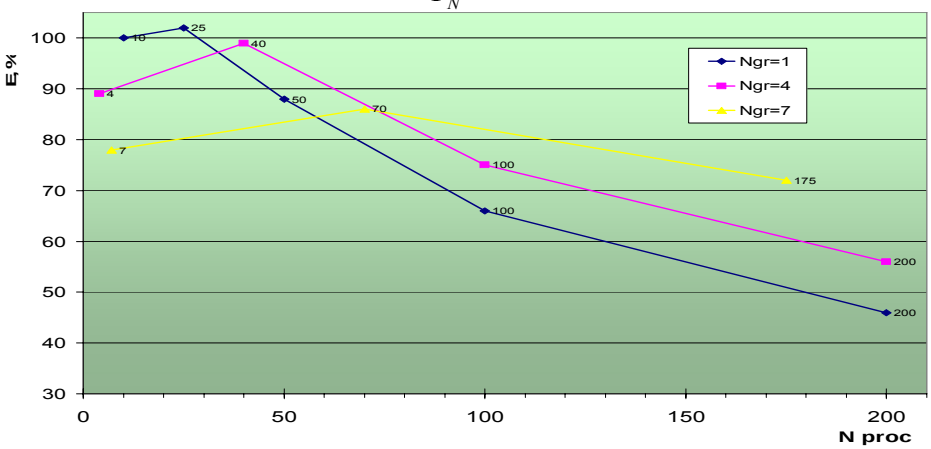


Figure 7. The speedup versus the number of processors.

To demonstrate capabilities of the developed methods, an example of computations for one demo problem (Figs.8 and 9) describing the experiments carried out on laser facility ISKRA (spark) is given below (Bel'kov et al., 2004).

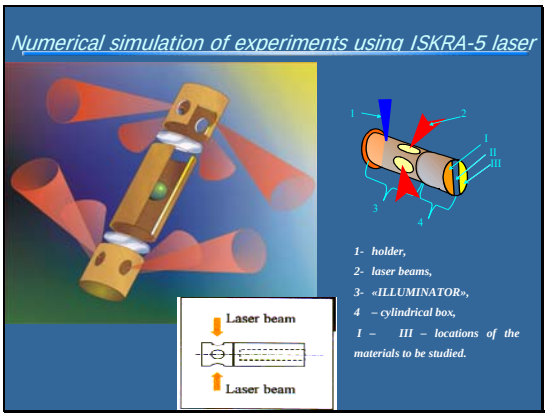


Figure 8. Statement of the problem on simulation of the experiment on laser facility ISKRA-5

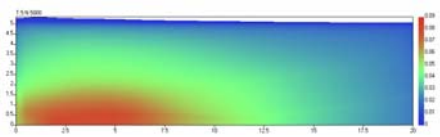
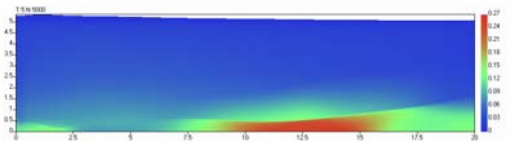
a)  b) 

Figure 9. The effective radiation temperature distribution (a) and the distribution of electrons (b) inside a cylinder at time 5 ns. The length scale - 100 μm, the temperature scale – KeV.

Conclusion

The transport equation is currently extensively used during simulation of various physical processes. Finding the numerical solutions to the problems of this category is accompanied with high computational costs. So, it's important to use numerical methods that allow a comprehensive consideration of the specific features of statements of the problems to be solved and the behavior of solutions of certain categories of application transport problems and provide, thereby, the required accuracy of computations in the most effective manner.

The paper presents an overview of numerical methods developed and used at RFNC-VNIIEF to solve multidimensional transport problems. These are the approaches based on Monte Carlo method, the method of angular coefficients (view factors) and grid approximations to the original transport equation using the method of finite difference and finite elements (deterministic grid methods). The paper briefly describes some specific features of the first two approaches. It contains a more detailed consideration of deterministic grid methods based on approximations to the transport equation using the finite difference and finite element methods with emphasis put on the methods algorithms and schemes for numerically solving multidimensional linked transport problems, in which, first, support of the required accuracy of computations using non-orthogonal spatial grids and, second, the efficiency of solving the transport equation on multiprocessors are of especial importance.

Acknowledgements

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References

Zhitnik A.K., Kochubey Yu.K., Subbotin A.N., et al. Use of Monte Carlo Method for Particle Transport Problems at RFNC-VNIIEF // Voprosy Atomnoy Nauki i Techniki. Ser.: Math. Model. Phys. Process. 1999. Iss.4. P. 64-67.

Kochubey Yu.K., Zhitnik A.K., Artemeva E.V., et al. C-95 Code. Monte Carlo Simulation of Joint Transport of Neutrons and Gammas. // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 2000. Iss. 2. P. 49-52.

Ivanov N.V., Ivanov A.N. Consideration of Thermal Atomic Motions, When Solving Neutron Transport Problems Using Monte Carlo. // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 2003. Iss. 4. P. 25-32.

Donskoy E.N. ELIZA Technique and Code for Solving Problems of Joint Transport of Photons, Electrons and Positrons With Monte Carlo Method // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1993. Iss. 1. P. 3-6.

Babayev Yu.N., Bazhenov S.V., Dementiev Yu.A. On Specific Features of the Solution to One Integral Radiation Transport Equation // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1978, Iss. 1(1), P. 7-9.

Zel'dovich Ya.B., Raiser Yu.P. Physics of Shock Waves and High-Temperature Hydrodynamic Effects. – Moscow, Nauka Publishers.1966. 688pp.

Dementiev Yu.A., Mashinin R.F., Nagornyi V.I, Rogachev V.G., Tikhomirova E.N. Numerical Technique for 3D Time-Dependent Problems of Energy Transport With Radiation in Vacuum // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1984. Iss. 2(16). P. 26-28.

Babayev Yu.N., Bazhenov S.V., Bazin A.A., et al. VNIIEF Techniques and Codes for 2D and 3D Problems of Time-Dependent Radiation Energy Transport in Regions of Complex Shapes Using View factors. Problems of Conversion Applications.// Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1995. Iss. 4. P. 3-8.

Bazin A.A., Skidan G.I., Tikhomorov B.P., Tikhomirova E.N. Numerical Simulation of Radiation Heat Exchange in Closed Multilayer Systems // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1998. Iss. 3. P. 34-40.

Bazin A.A., Vatulin V.V., Dementiev Yu.A., Mironova V.F., Skidan G.I., Tikhomirov B.P., Tikhomirova E.N. Application of View Factor Method in Calculations of Radiation Transport in Targets. Nuclear Instruments and Methods in Physics Research, A415 (1998), 123-126.

Dementiev Y.A., Mashinin R.F., Mironova V.F., Chentsov N.N. Approximate Computation of Angular Distribution Factors for Time-Dependent radiation of Heat From Rotation Surface // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1983, Iss. 1(12), P.32-34.

Shagaliev R.M., Shumilin V.A., Alekseyev A.V., Belyakov I.M., Evdokimov V.V., et al. Numerical Models and Techniques for Solving Multidimensional Particle and Energy Transport Problems Implemented in SATURN-3 Code Complex// Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1999, Iss.4 P. 20-26.

Pleteneva N.P., Shagaliev R.M. Approximation to the 2D Transport Equation With The Extended Template Scheme Using Quadrangular and Polygonal Grids // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1989. Iss. 3, P. 34-41.

Troshchiyev V.E. On categories of Grids Allowing Conservative Approximations to the 2D Transport Operator With the Triangular Difference Operator. // Journal of Computational Mathematics and Mathematical Physics. 1976. V. 16, No. 3. P. 793-797.

Moskvin A.N., Shumilin V.A. A Technique for Numerical Solution of the 2D Transport Equation Using Irregular Polygonal Grids // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 2005. Iss. 1. P.31- 40.

Shagaliev R.M. An Adaptive Method for Solving a Multidimensional Transport Equation Using Refined Grids in Phase Space // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 2004. Iss. 4, P. 3-15.

Shagaliev R.M., Alekseyev A.V., Belyakov I.M., Gichuk A.V., Evdokimov V.V., Moskvin A.N., Nuzhdin A.A., Pleteneva N.P., Shemyakina T.V. Numerical Solution of the 2D Transport Equation Using the Space-Adaptive Method of Refined Grids // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 2004. Iss. 4, P 16-25.

Fedotova L.P., Shagaliev R.M. The Finite-Difference Method KM for 2D Time-Dependent Transport Processes in Multigroup Transport Approximation. Mathematical Modeling. V. 3. No.6, 1991.

Evdokimov V.V., Shagaliev R.M. FCA Method for Acceleration of Iteration Convergence During Numerical Solution of the Neutron Transport Equation Using Non-Orthogonal Grids. // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1994. Iss. 3. P.11-17.

Evdokimov V.V., Shagaliev R.M. Some Results of Numerical Studies of FCA Method, When Solving 2D Transport Problems Using DSn-type Schemes. Algorithms and Codes for Neutron-Physical Computations of Nuclear Reactors. “Neutronics-94”. Obninsk 1996. P.108-112..

Moskvin A.N., Nikiforova A.V., Shagaliev R.M., et al. EKRAN Code for the Calculation of the Effective Neutron Multiplication Factor for Reactor Systems, Proceeding of the International Conference on Reactor Physics, 16-20 September, 1996,, Japan, V.3, P.A386-A395.

Alekseyev A.V., Nuzhdin A.A., Shagaliev R.M. Fine-Grain Parallelization Method for 2D and 3D Transport Problems Using Non-Orthogonal Grids.// Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 2001. Iss.2,P. 3-11.

Alekseyev A.V., Shagaliev R.M. Parallelization Algorithm for Numerical Solution of the 3D Transport Equation in SATURN Complex // Voprosy Atomnoy Nauki I Techniki. Ser.: Math. Model. Phys. Process. 1993. Iss. 3. P. 3-7.

Bel'kov S.A., Belyakov I.M., Nuzhdina I.S., Vinokurov O.A. Computations of the Radiation Field in HOHLRAUM Type Targets under Conditions of Experiments on ISKRA-5 Facility. Proceeding of the 31st Conference on Plasma Physics of European Physical Community, London, 28 June –2 July, 2004. Volume 28G.