# Institute of Theoretical and Mathematical Physics

*Russian Federal Nuclear Center -*

*VNIIEF*

# The Effect of Viscosity, Dispersion, and Phase Transition Kinetics on the Parameters of Rarefaction Shock Waves for Non-Convex Equations of State


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# Abstract


- *It has been analytically proved that consideration of various physical processes, such as viscosity, dispersion, or kinetics of phase transitions, leads to different parameters of rarefaction shock waves emerging in materials having non-convex equations of state.*
- *One-dimensional simulations confirm these results.*
- *Hence, the problem of selecting a single and "correct" rarefaction shock wave should be solved at the level of physical model selection and depends on major physical processes to be ignored, when writing the ideal gas dynamics equations.*

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# Introduction. Admissibility by the vanishing viscosity method - 1

- Equation of state $p=P_0(v)$ is smooth enough, strictly monotonous and has only one section of "reverse" convexity

$$\begin{aligned} &\frac{d^2V_0(p)}{dp^2} > 0, \quad \text{åñëè} \quad p < p_{min} \quad \text{èëè} \quad p > p_{max}; \\ &\frac{d^2V_0(p)}{dp^2} < 0, \quad \text{åñëè} \quad p_{min} < p < p_{max}; \end{aligned} \qquad (1)$$

Two Renckin-Hugoniot requirements $\Rightarrow$ the mass velocity of the rarefaction shock wave is

$$\sigma = \sqrt{\frac{p_1 - p_2}{V_0(p_2) - V_0(p_1)}} \qquad (2)$$

The rarefaction shock wave $(p_1,v_1) \to (p_2,v_2)$ in the limit self-similar solution should meet the requirements

$$p_2 \le p_{min} < p_{max} \le p_1 \qquad (3)$$

$$(s(p_2))^2 \equiv \left( -\frac{dV_0(p)}{dp}\bigg|_{p=p_2} \right)^{-1} \le \sigma^2 \equiv \frac{p_1 - p_2}{V_0(p_2) - V_0(p_1)} \le (s(p_1))^2 \equiv \left( -\frac{dV_0(p)}{dp}\bigg|_{p=p_1} \right)^{-1} \qquad (4)$$

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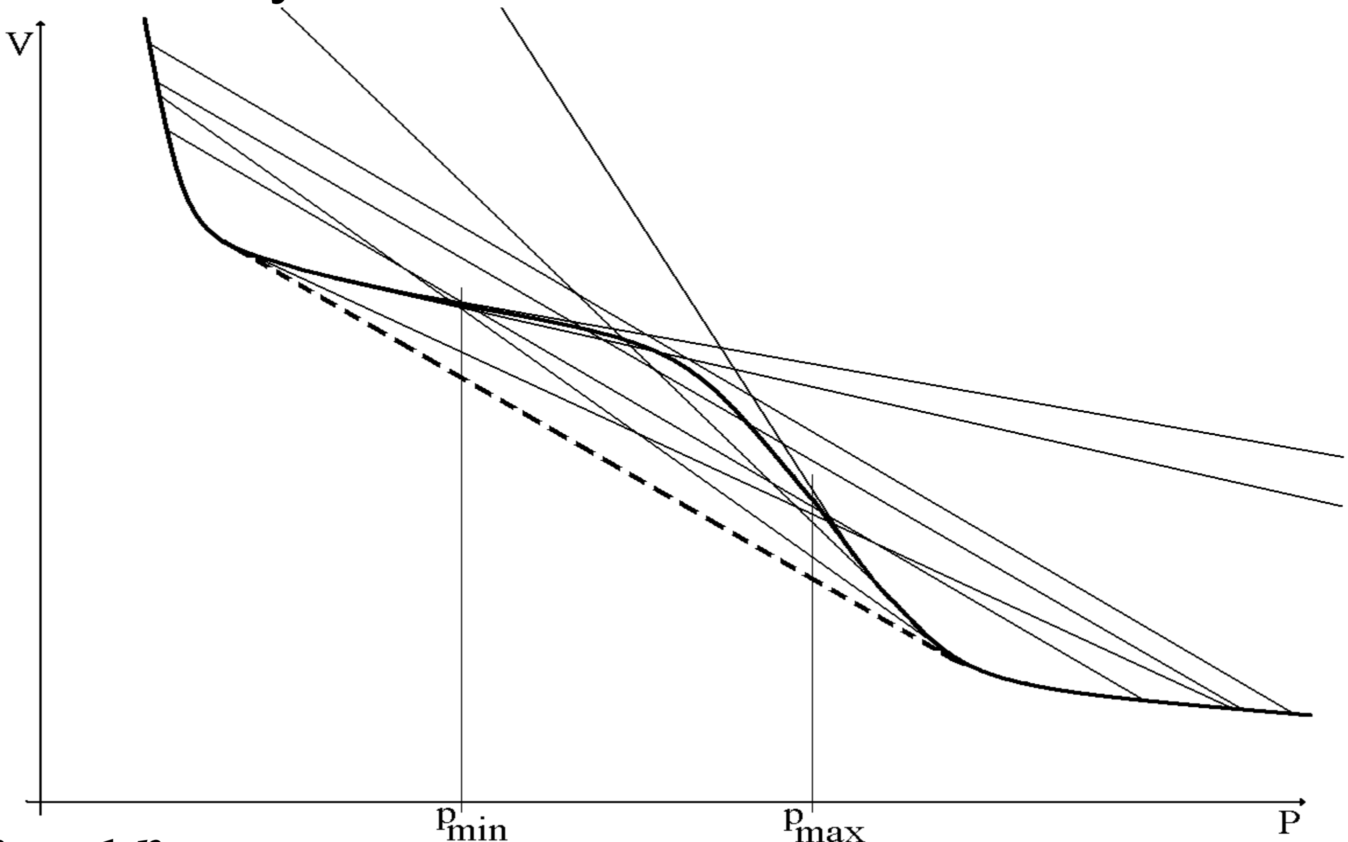
# Introduction. Admissibility by the vanishing viscosity method - 2

- Possible variants of positions of Rayleigh-Michelson straight-line segment $v=V_{RM}(p)$ for rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ meeting the necessary requirements (3) and (4).

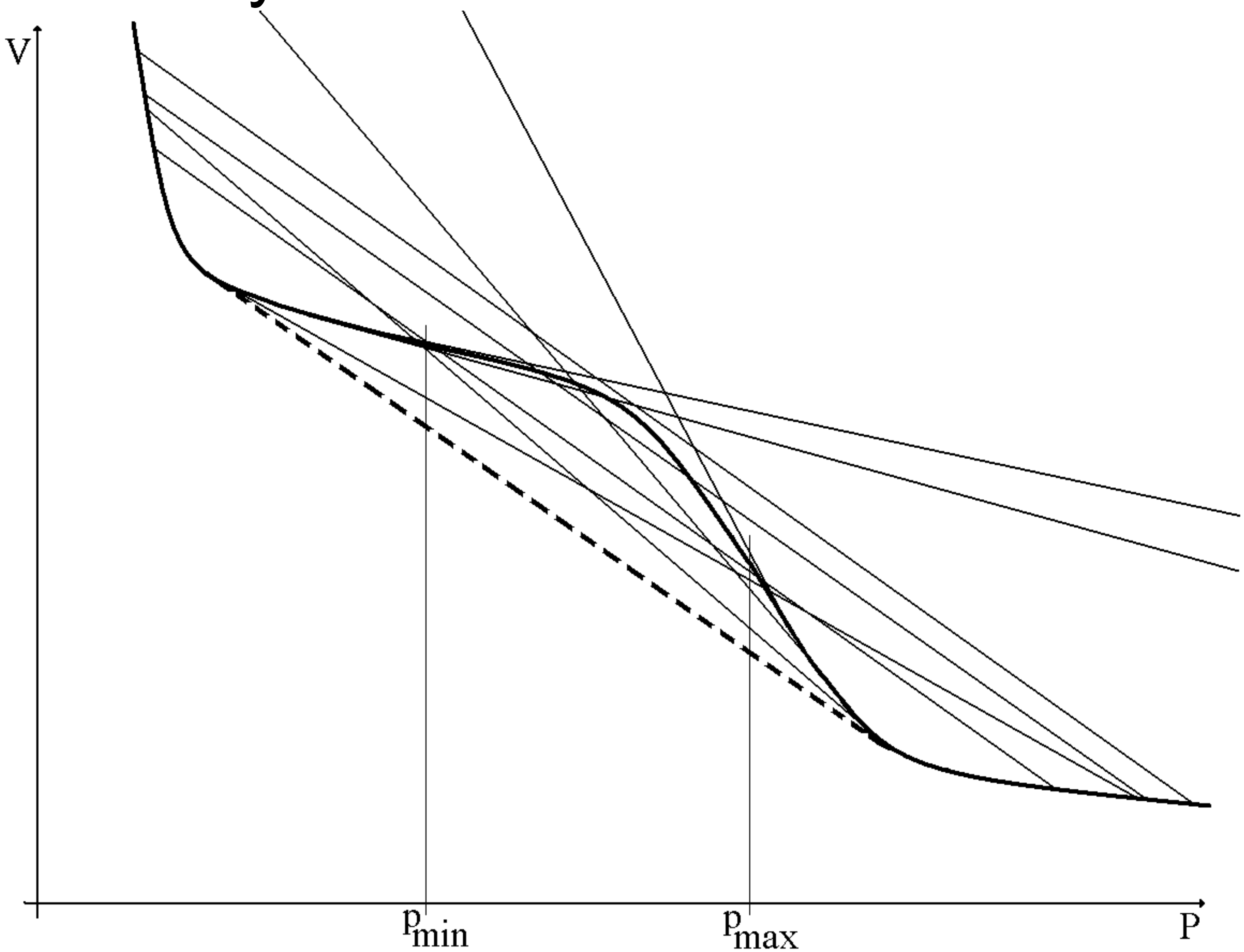



(3) $$p_2 \le p_{min} < p_{max} \le p_1$$

(4) $$\left(s(p_2)\right)^2 \equiv \left(-\left.\frac{dV_0(p)}{dp}\right|_{p=p_2}\right)^{-1} \le \sigma^2 \equiv \frac{p_1-p_2}{V_0(p_2)-V_0(p_1)} \le \left(s(p_1)\right)^2 \equiv \left(-\left.\frac{dV_0(p)}{dp}\right|_{p=p_1}\right)^{-1}$$

# Introduction. Admissibility by the vanishing viscosity method - 3

- An impact transition $(p_1,v_1,u_1)\rightarrow(p_2,v_2,u_2)$ is considered admissible, if there is a classic solution to the gas dynamic equation system for viscous gas with any positive coefficient of viscosity $\mu>0$ that connects points $(p_1,v_1,u_1)$ and $(p_2,v_2,u_2)$. Such impact transition is said to be admissible by the vanishing viscosity method: Galin, G.Ya., "On the Theory of Shock Waves," DAN USSR, **127**, 55-58 (1959).
- It is the dotted line.



# The Method of Vanishing Normal Dispersion - 1

- Consider the small normal dispersion ($\eta$=const, $\eta \to +0$) in 1D gas dynamics equations in Lagrangian variables

$$(6) \qquad \frac{dv}{dt} - \frac{\partial u}{\partial m} = 0 \qquad\qquad (7) \qquad \frac{du}{dt} + \frac{\partial p}{\partial m} - \eta \frac{\partial^3 v}{dt^2 \partial m} = 0$$

We say that the rarefaction shock wave $(p_1, v_1, u_1) \to (p_2, v_2, u_2)$ is admissible by normal dispersion (or admissible by the method of normal dispersion), if there is a classic solution to the system of gas dynamics equations 5 and 7 with normal dispersion and with any positive value of dispersion coefficient $\eta > 0$ that connects points $(p_1, v_1, u_1)$ and $(p_2, v_2, u_2)$.

Consider the structure of time-independent shock wave and determine conditions under which equations 6 and 7 have continuous solutions of the form

$$\vec{U}(t,m) = \vec{U}(\mu); \quad \vec{U} = (u, P, v); \quad \mu = m - \sigma \cdot t; \quad \sigma = const > 0$$

$$\vec{U}(\mu) \to \vec{U}_2, \ \frac{\partial \vec{U}}{\partial \mu} \to 0 \ \ at \ \ \mu \to -\infty \qquad\qquad \vec{U}(\mu) \to \vec{U}_1, \ \frac{\partial \vec{U}}{\partial \mu} \to 0 \ \ at \ \ \mu \to \infty$$

# The Method of Vanishing Normal Dispersion - 2

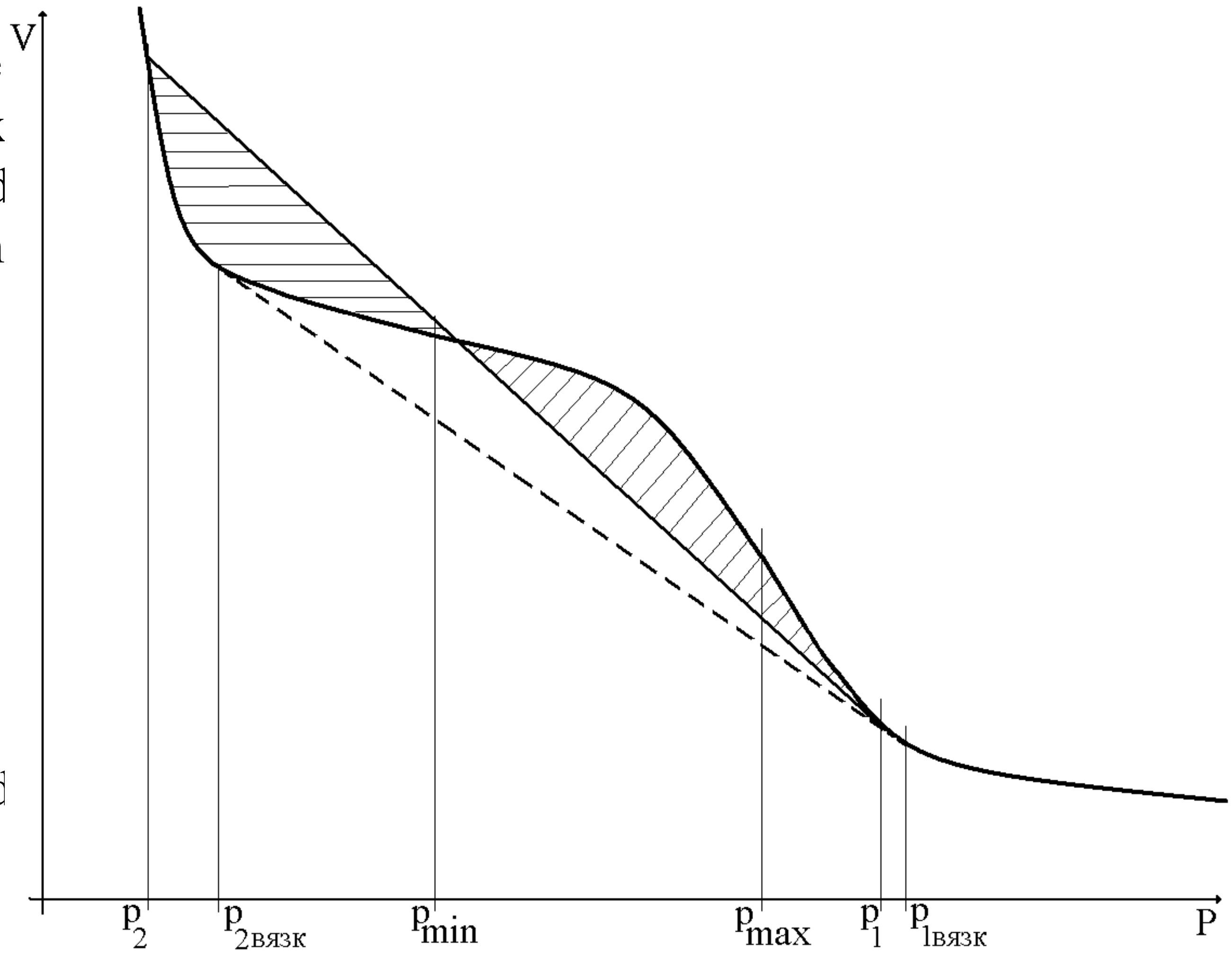
- **Theorem 1**. *In the self-similar Riemann problem with non-convex equation of state there is a rarefaction shock wave admissible by the method of vanishing normal dispersion. This rarefaction shock wave is the single one uniquely determined by the following features. First, in plane (P,V) the segment of Rayleigh-Michelson straight line* $v=V_{RM}(p)$ *of the rarefaction shock wave* $(p_1,v_1)\rightarrow(p_2,v_2)$ *is tangent to the equation of state* $v=V_0(p)$ *at its end point* $(v_1,p_1)$ *and, secondly, the Rayleigh-Michelson straight line intersects the adiabatic curve* $v=V_0(p)$ *at an intermediate point and at the end point* $(p_2,v_2)$*, so that the integral for* $v=v_2$ *equals zero:*

$$\Phi(v)=\int_{v_1}^{v}\left[P_0(v)-\frac{p_1\cdot\left(v_2-v\right)+p_2\cdot(v-v_1)}{v_2-v_1}\right]dv>0,\quad \forall v: v_1<v<v_2$$

$$\int_{v_1}^{v_2}\left[P_0(v)-\frac{p_1\cdot\left(v_2-v\right)+p_2\cdot(v-v_1)}{v_2-v_1}\right]dv=0$$

# The Method of Vanishing Normal Dispersion - 3

- The adiabatic curve with the non-convex equation of state and Rayleigh-Michelson straight lines of the rarefaction shock wave admissible by the method of vanishing normal dispersion ($p_1 \rightarrow p_2$) and the method of vanishing viscosity ($p_{1вязк} \rightarrow p_{2вязк}$, dotted line).



# The Method of Artificial Interphase Kinetics - 1

- Assumed that a material consists of two mixed phases ***with convex equations of state*** $v=V_1(p)$ and $v=V_2(p)$ ***for each of the pure phases***, the equilibrium equation of state $v=V_0(p)\equiv\Theta_0(p)\cdot V_1(p)+(1-\Theta_0(p))\cdot V_2(p)$ is strictly monotonous, however, it has one non-empty range of non-convexity, where $\Theta_0(p)$ is a strictly monotonous equilibrium concentration of the first phase.

- In the artificial interphase kinetics method the non-equilibrium equation of state $v=V(p,\Theta)\equiv\Theta\cdot V_1(p)+(1-\Theta)\cdot V_2(p)$ is used and the concentration $\Theta(t,m)$ of the first phase is described by the non-equilibrium equation of the phase transition kinetics

$$\tau\frac{d\Theta}{dt}=-\left(\Theta-\Theta_0(p)\right)\cdot\omega(\Theta,p)$$

with positive time of phase relaxation $\tau=const\to+0$ and positive function $\omega(\Theta,p)$ continuous over the set of arguments, which describes the dependence of the “rate” of interphase relaxation on pressure and phase concentrations. Viscosity and dispersion are not taken into consideration.

Akhmadeyev, N.Kh., Akhmetova, N.A., Nigmatulin, R.I. "The Structure of Shock-Wave Flows with Phase Transitions in Fe Near the Free Surface," *Journal of Applied Mathematics and Engineering Physics.* No.6, 113-119 (1984)

# The Method of Artificial Interphase Kinetics - 2
# Case of not intersected EoS of pure phases

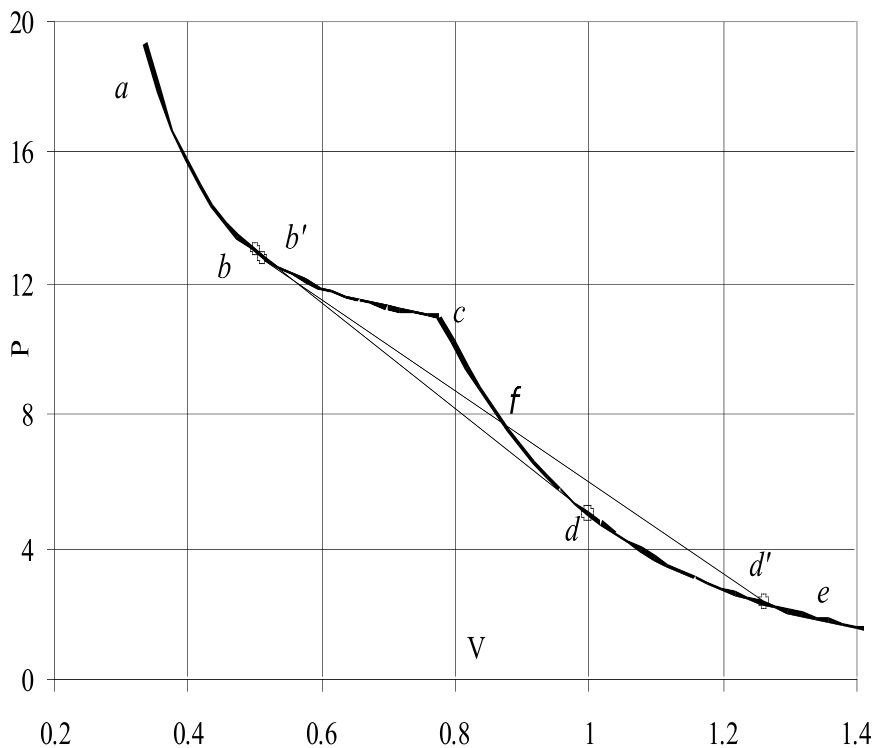
- Consider a case of not intersecting equations of state of two pure phases, $v=V_1(p)$ and $v=V_2(p)$: $V_2(p)>V_1(p)$.
- **Theorem 2**. *Let the equations of state of two pure phases $v=V_1(p)$ and $v=V_2(p)$ be not intersected. Then, there is a rarefaction shock wave obtained using the method of artificial interphase kinetics. This is a single rarefaction wave , which is uniquely determined by the following feature. In plane (P,V) the Rayleigh-Michelson straight-line segment $v=V_{RM}(p)$ of the rarefaction shock wave $(p_1,v_1)\to(p_2,v_2)$ is tangent to the equilibrium equation of state $v=V_0(p)$ at its end points $(v_2,p_2)$ and $(v_1,p_1)$.*
- The parameters of the rarefaction shock wave obtained using the method of artificial interphase kinetics are the same as the parameters of the rarefaction wave obtained for the equilibrium equation of state $v=V_0(p)$ with the method of vanishing viscosity.
- Bondarenko, Yu.A., Sofronov, V.N. " Rarefaction Shock Wave Non-Uniqueness: the Role of Interphase Kinetics," *Voprosy Atomnoy Nauki I Tekhniki. Ser. Math. Model. Phys. Process*. Issue 1, 28-46 (2004).

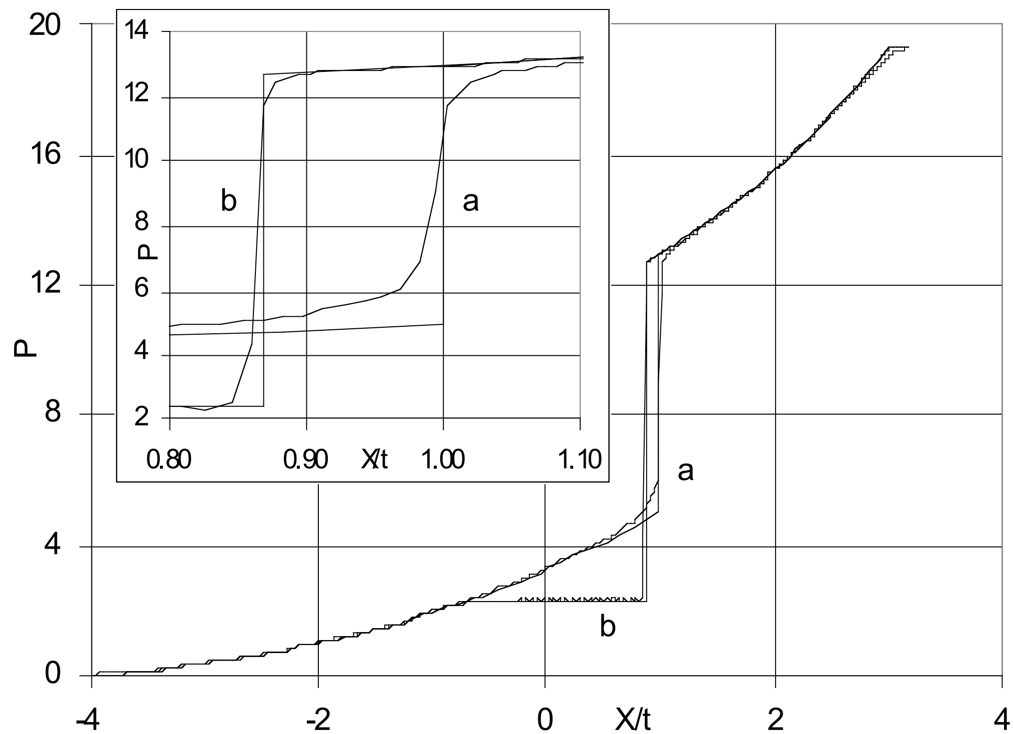
# The Method of Artificial Interphase Kinetics - 3
# Case of intersected EoS of pure phases

- Consider the method of artificial interphase kinetics for a case, when the equations of state of two pure phases, $v=V_1(p)$ and $v=V_2(p)$ intersect at a single point $(p_3,v_3)$, $p_1>p_3>p_2$ and assume that $P_2(v)>P_1(v)$ at $v<v_3$, and $P_2(v)<P_1(v)$ at $v>v_3$.
- **Theorem 3**. *Let the equations of state of two pure phases $v=V_1(p)$ and $v=V_2(p)$ intersect at a single point $(p_3,v_3)$. Then, there is a rarefaction shock wave obtained using the method of artificial interphase kinetics in the self-similar Riemann problem. This rarefaction shock wave is a single one and it is uniquely determined by the following feature. In plane $(P,V)$ the Rayleigh-Michelson straight-line segment $v=V_{RM}(p)$ of the rarefaction shock wave $(p_1,v_1)\rightarrow(p_2,v_2)$ passes through the phase intersection point $(p_3,v_3)$, is tangent (from below) to the equilibrium equation of state $v=V_0(p)$ at one of its end points, $(v_1,p_1)$ and intersects (from above and with no contacts) the equilibrium equation of state at the second of its end points, $(p_2,v_2)$.*
- In case of intersected equations of state of pure phases, the parameters of the rarefaction shock wave obtained using the method of artificial interphase kinetics significantly differ from that of the rarefaction wave obtained for the equilibrium equation of state $v=V_0(p)$ using the method of vanishing viscosity and method of vanishing dispersion.

# 1D Computations Rarefaction Shock Waves - 1

- **Task 1**: $c(\rho)=b_k\rho$ at $\rho_{k-1}<\rho<\rho_k$, $b_1=4.0$, $b_2=1.0$, $\rho_1=(32/15)^{1/3}$.
- Computations with viscosity (a) and dispersion (b)



# 1D Computations Rarefaction Shock Waves - 2

- The interphase kinetic equation of the form

$$\frac{d\Theta}{dt} = j_{21} - j_{12} = \frac{V_2(p)\cdot f_{21}(p)\cdot(1-\Theta) - V_1(p)\cdot f_{12}(p)\cdot\Theta}{\tau}$$

$$f_{12} = \begin{cases} 1 - exp\left(-\left(\dfrac{p - p_{12}}{\Delta_{12}}\right)^3\right) & p > p_{12} \\ 0 & p < p_{12} \end{cases} \qquad f_{21} = \begin{cases} 1 - exp\left(-\left(\dfrac{p_{21} - p}{\Delta_{21}}\right)^3\right) & p < p_{21} \\ 0 & p > p_{21} \end{cases}$$

Akhmadeyev, N.Kh., Akhmetova, N.A., Nigmatulin, R.I. "The Structure of Shock-Wave Flows with Phase Transitions in Fe Near the Free Surface," *Journal of Applied Mathematics and Engineering Physics.* No.6, 113-119 (1984).

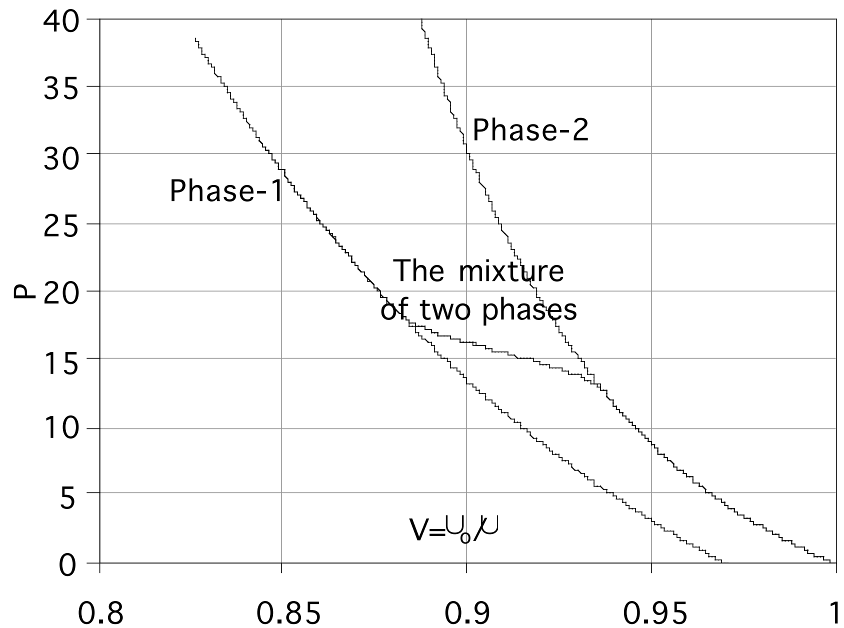
The equation of state:

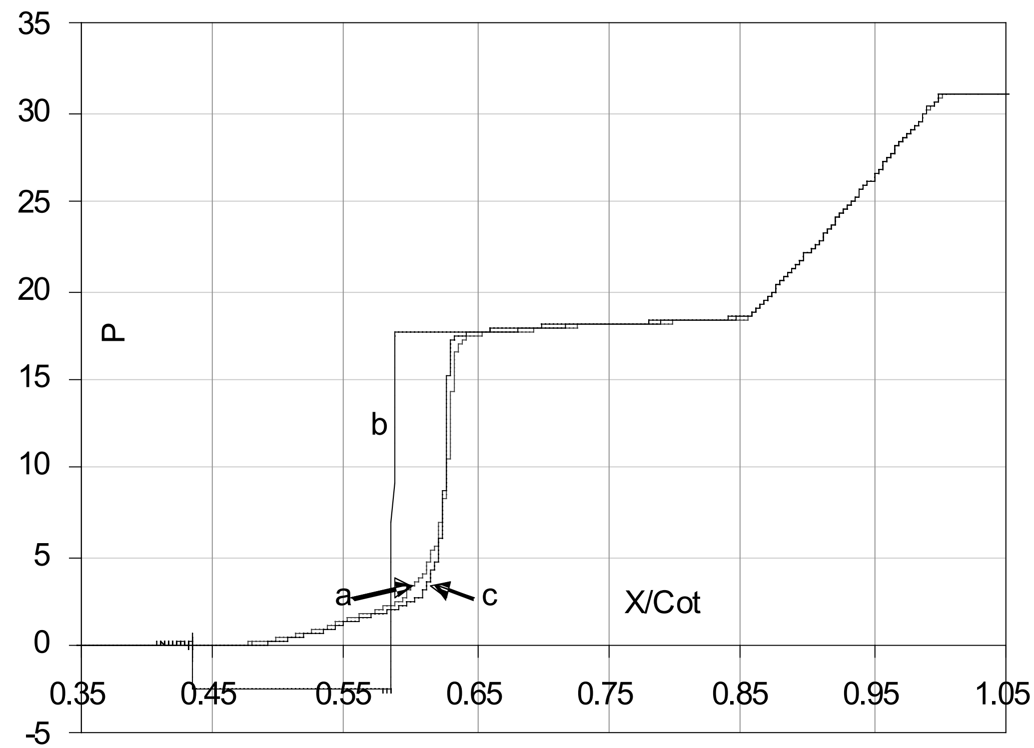
$$\frac{\rho_0}{\rho_i} = V_i(P) = \left(\frac{a_i}{P - b_i}\right)^{n_i}, i = 1,2$$

$$\frac{\rho_0}{\rho} = V = \Theta\cdot V_1(P) + (1-\Theta)\cdot V_2(P)$$

# 1D Computations Rarefaction Shock Waves - 3

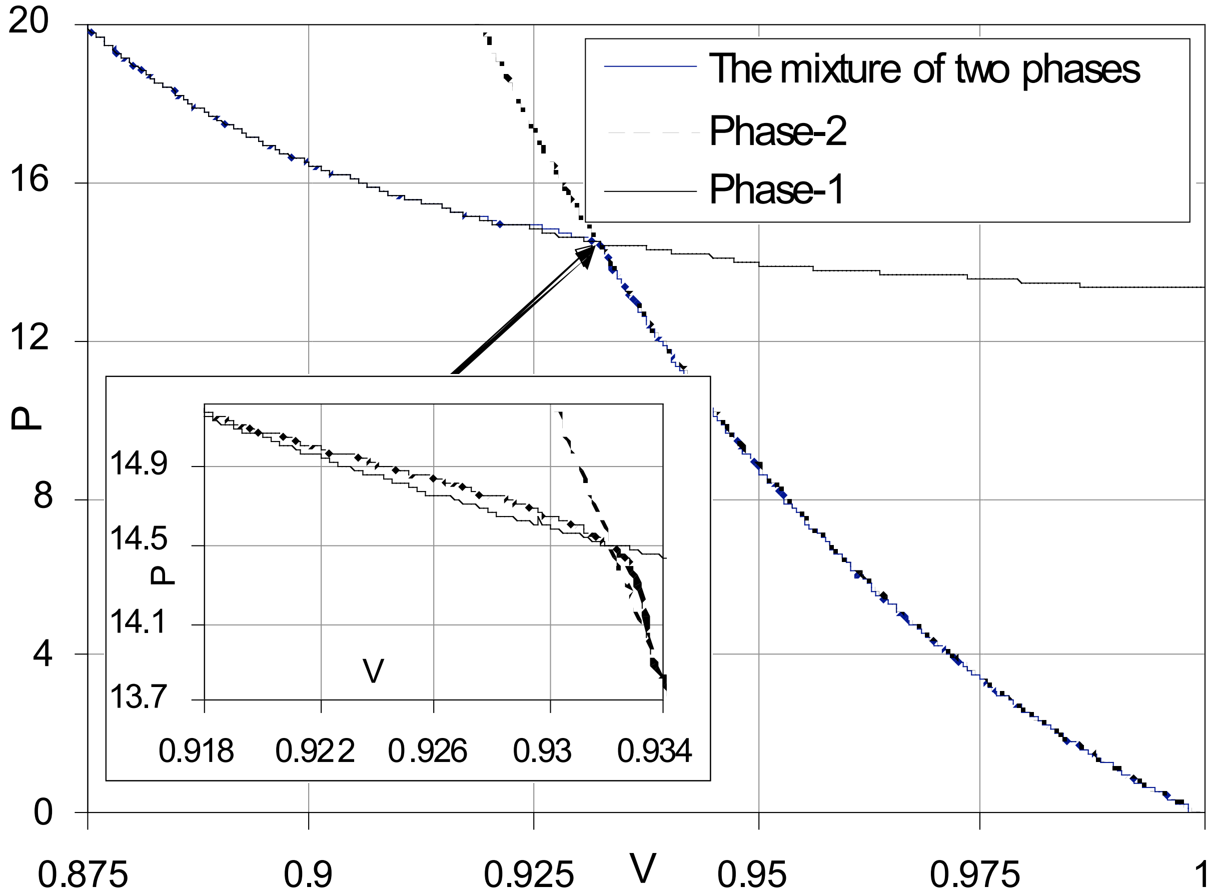
- **Task 2**. The mixture of two phases without intersection of state equations of pure phases.
- Computations with viscosity (a), dispersion (b), and interphase relaxation (c).



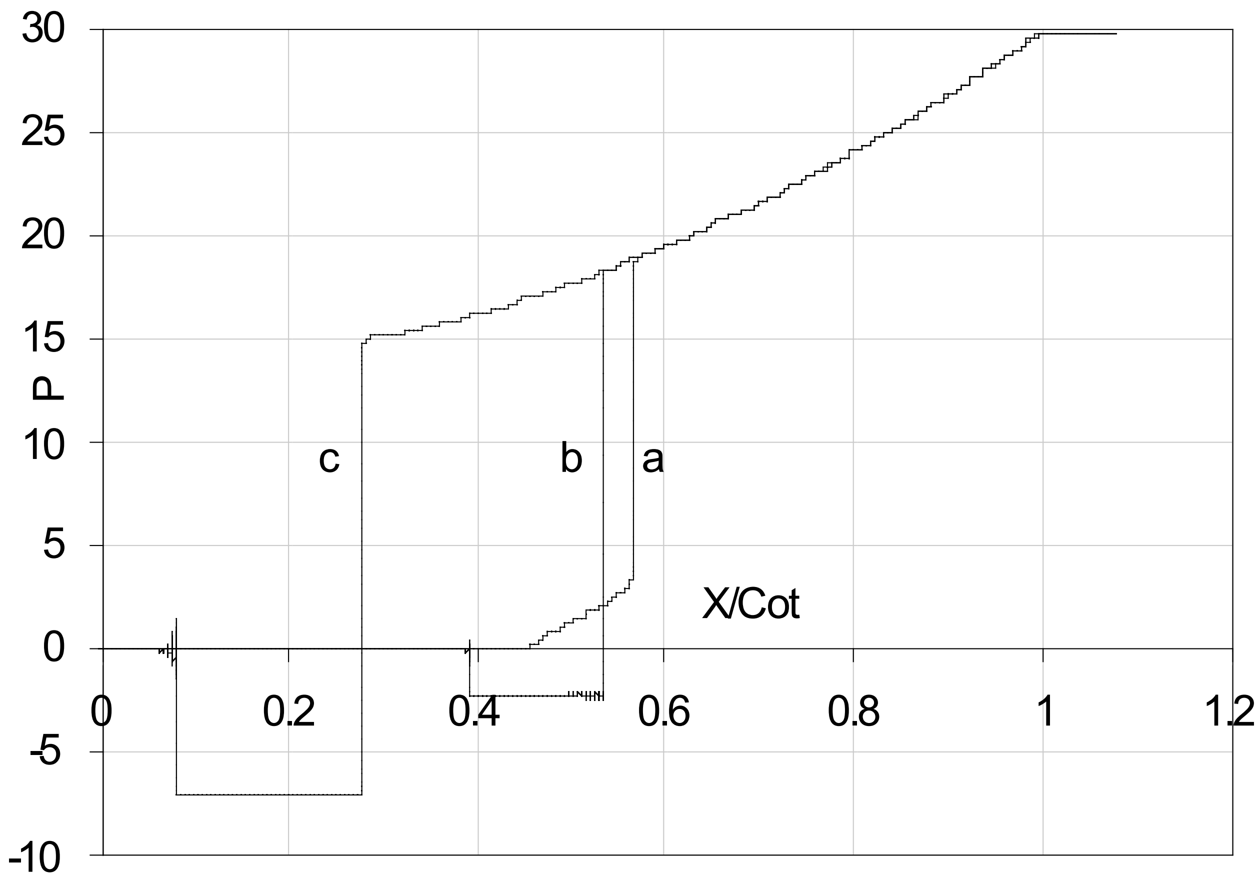
# 1D Computations Rarefaction Shock Waves - 4

- **Task 3**. The mixture of two phases with intersection of EOSes of the pure phase .
- The two-phase equation of state with intersection of phases:



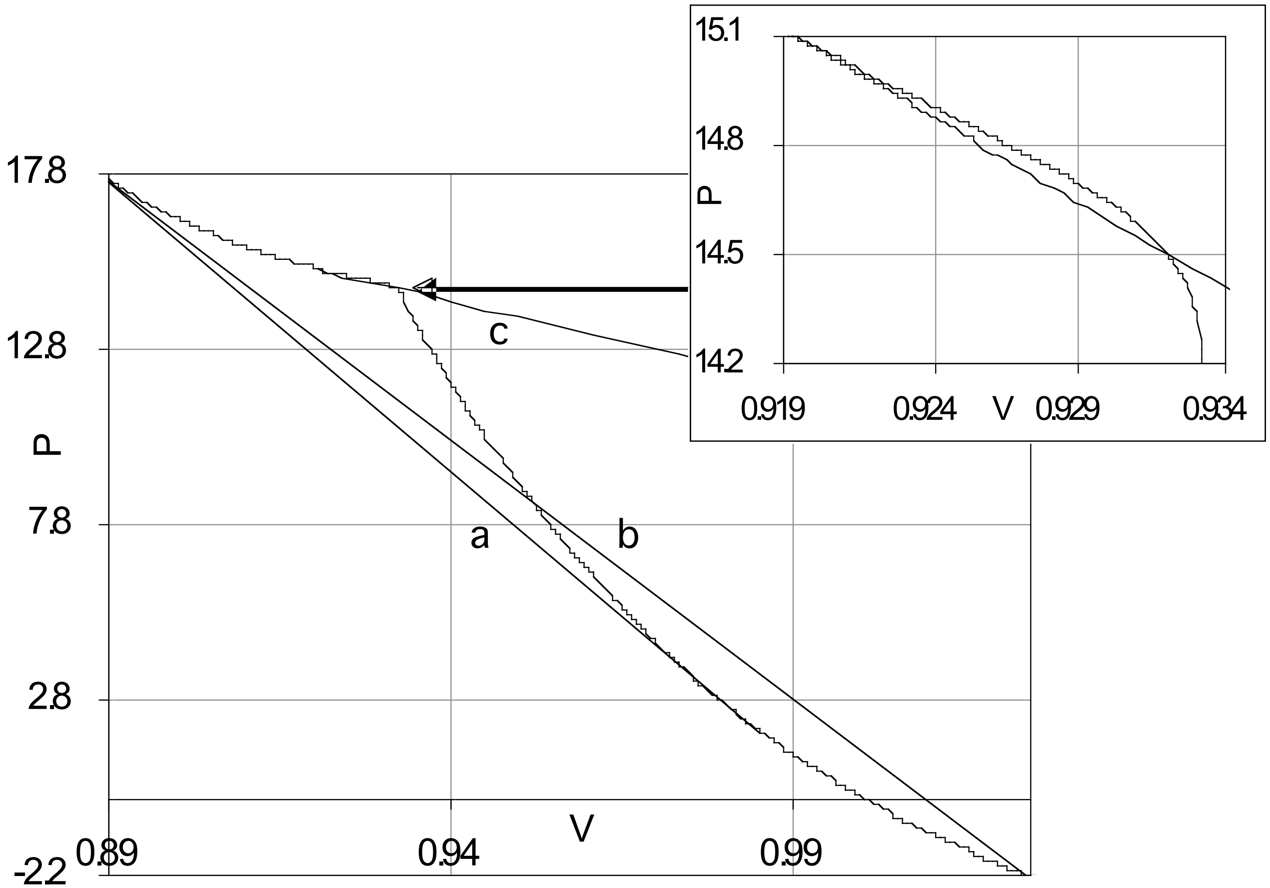
# 1D Computations Rarefaction Shock Waves - 5

- **Task 3**. The mixture of two phases with intersection of EOSes of the pure phase .
- Computations with viscosity (a), dispersion (b), and interphase relaxation (c):



# 1D Computations Rarefaction Shock Waves - 6

- **Task 3**. The mixture of two phases with intersection of EOSes of the pure phase .
- The shock wave position in task 3 computations with viscosity (a), dispersion (b, and interphase relaxation (c) ):



# Conclusion

- The results obtained in the paper prove that parameters of rarefaction shock waves depend on the processes inside the smeared rarefaction shock wave front.
- The results allow a unequivocal conclusion that the problem of choosing a single and "correct" rarefaction shock wave during simulation (including numerical simulation) of gas dynamic processes should be solved at the level of selecting physical models and depends on the major physical processes to be ignored, when writing the ideal gas dynamics equations.
- This conclusion is not new, in general. The review paper by Kulikovsky (1988) gives many examples of such a kind:

Kulikovsky, A.G. "Severe Discontinuities in Continuum Flows and Their Structure," *Works of V.A.Steklov Institute of Mathematics of the USSR Academy of Sciences*. **182**. 261-291 (1988).