RAMZES-CP Technique for Solving of Spatial Multimaterial Heat Conduction Flows in Eulerian-Lagrangian Coordinates

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Plan of presentation

1. Introduction into RAMZES-CP technique
2. Movable and immovable grids
3. Combined grids
Introduction

RAMZES-CP technique is based on the following principles:

• using the gas dynamics equations and heat conduction equation written both in the Cartesian and curvilinear coordinate systems, in Eulerian-Lagrangian variables;

• using the implicit finite difference approximation in time both to the heat conduction equation and the gas dynamics equations;

• splitting into physical processes;

• partition of the problem geometry to fragments, in each of which its most appropriate grid is constructed;

• the resultant system of multidimensional finite difference equations is solved using the splitting into directions and the direct method for solution of finite difference equation subsystem on block-matrix type grids, i.e. the integrated sweep method, which has been developed by the authors;

• the method of concentrations and Youngs method are used for construction of interfaces in the computations of material flows from mixed cells

• the parallel computations are employed at all phases (preprocessing, computation, and analysis of results) of the task run on the multi-processor distributed-memory computer.
Equations of multi-component gas dynamics

\[ \frac{d\rho_i}{dt} = -\rho_i \cdot \text{div} \bar{U}, \quad i = 1, \ldots, N \]

\[ \frac{d\bar{U}}{dt} = -\frac{1}{\rho} \cdot \text{grad} P \]

\[ \frac{d\varepsilon_i}{dt} = -\frac{P_i}{\rho_i} \cdot \text{div} \bar{U}, \quad i = 1, \ldots, N \]

\[ P_i = P_i(\rho_i, T), \quad i = 1, \ldots, N \]

\[ \varepsilon_i = \varepsilon_i(\rho_i, T), \quad i = 1, \ldots, N \]

\[ \rho = \sum_{i=1}^{N} \rho_i \cdot \xi_i \]

\[ P = \sum_{i=1}^{N} P_i(\rho_i, \varepsilon_i) \xi_i \]

\[ \frac{d\xi_i}{dt} = 0, \quad i = 1, \ldots, N \]

- \( \rho_i \) - component density,
- \( \varepsilon_i \) - component specific internal energy,
- \( \xi_i \) - component volume concentration,
- \( P_i \) - component pressure,
- \( N \) - number of components in the mixed cell,
- \( \rho \) - cell density,
- \( P \) - cell pressure,
- \( U \) - cell velocity vector,
- \( T \) - cell temperature.
Curvilinear coordinate system

\[ X = (r \cdot \sin \theta (S) + R_0 (S)) \cdot \cos \varphi \]
\[ Y = (r \cdot \sin \theta (S) + R_0 (S)) \cdot \sin \varphi \]
\[ Z = r \cdot \cos \theta (S) + Z_0 (S) \]
Centering of variables

\[ (r, U)_{m+1, k, l} \]

\[ V_{m+1/2, k-1/2, l} \]

\[ (T, P, e, \rho)_{m+1/2, k, l} \]

\[ W_{m+1/2, k+1/2, l} \]

\[ W_{m+1/2, k, l-1/2} \]
Solution domain decomposition over processors

Line

Columns
Parallel-pipeline method for 4 processors (P1-P4) at 4 time moments (t1-t4)
Parallel-pipeline method

Run time dependency on the number of portions and number of processors
Movable and immovable grids.

Movable grids

- material interfaces are tracked automatically
- the computation cost-efficiency in terms of the number of needed grid nodes and computation amount
- the computation results are highly accurate
- cell overlaps leading to crashes are possible

Immovable grids

- the need for tracking material interfaces not connected to the grid
- the initial grid quality preservation

It is clear that neither purely Lagrangian, nor purely Eulerian computation method is ideally suitable for many problems, while some combination of the Eulerian and Lagrangian approximations can be the best computational method. Experience suggests that in most problems the computational domain can be divided into subdomains, so that for large time intervals each subdomain can be best approximated by the movable and immovable grids.
Problem of adiabatic expansion of triaxial gas ellipsoid into vacuum (movable grid)

Initial time

Final time
Movable and immovable grids.

Movable grids

+ material interfaces are tracked automatically
+ the computation cost-efficiency in terms of the number of needed grid nodes and computation amount
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Immovable grids

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Rotating and Moving Jack - 3D (immovable grid)
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Initial geometry of problem

One full turn (200 steps)

Grid 40*40*40
Movable and immovable grids.

**Movable grids**
- material interfaces are tracked automatically
- the computation cost-efficiency in terms of the number of needed grid nodes and computation amount
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**Immovable grids**
- the need for tracking material interfaces not connected to the grid
- the initial grid quality preservation

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Combined grid – grid at initial time
Combined grids – groups of cells
Combined grid - pressure

\[ P = \frac{P1 \times AC + P2 \times CD + P3 \times DE + P4 \times EB}{AB} \]
Combined grids
Combined grid - pressure
Combined grids - comparison

Combined grids vs. Lagrangian grids
Conclusion

The presentation has discussed the technique RAMZES-KP designed for computing multi-component heat-conducting flows in Eulerian-Lagrangian coordinates on parallel distributed-memory computer systems.

Also, the modern directions of further development of the technique are presented.