Relaxation Model of Shear Strength of Five Metals (Aluminum, Beryllium, Copper, Tantalum, Depleted Uranium)

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Summary: In the paper, the authors present constitutive relations for five metals, namely, aluminum alloy AMg6, beryllium, copper M1, tantalum, depleted uranium (DU). The model takes account for strain hardening and compression hardening, thermal softening, influence of strain rate, transition from sliding to twinning under high strain rate conditions.

Introduction

Scientists of various countries have been involved in development of models of material behavior at high strain rates since 70-80-th of the previous century. However, though there are such a history and achievements of present-day numerical methods, development of equations for determination of relations between stress tensor and strain tensor is still an urgent unsolved problem. First of all, it is caused by complexity of the deformation process. It manifests itself in competition of various strain mechanisms, occurred structural changes, violation of flow stability at various scale levels, etc. On the one hand, the models for calculations of high-rate dynamic processes should be rather simple and physically clear, and on the other hand, they should cover the main processes, which occur in materials, and they should "work" in a wide range of flow parameters. The authors present a relaxation elastic-plastic model of strength for five metals, which are aluminum alloy AMg6, beryllium, copper, tantalum and depleted uranium. The suggested model can be used in calculations of complex dynamic processes, which are accompanied with high strain rate, compression hardening and strain hardening.

Basic r\Relations

For calculations of deviatoric component of shear stress, the following phenomenological elastic-plastic relaxation model is used:

$$\dot{Y}_d = 3G\dot{e}_i - \frac{Y_d - Y_S}{t}, \qquad Y_S = Y_0 \times \left[1 + a_0 \times \left(1 - \bar{T}^k\right) \times \left(1 - \exp\left(-\frac{e_i^p}{e_1}\right)\right)\right] \times \frac{G}{G_0},$$

$$t = \frac{t_0 \times \left(1 - \bar{T}\right)^2}{\frac{G}{G_0} \times \left(1 + \frac{\dot{e}_i}{\dot{e}_{i0}} \times j\left(\dot{e}_i\right)\right)^{0.5}}, \qquad j\left(\dot{e}_i\right) = \frac{1 + j_{\max} \times \left(\frac{\dot{e}_i}{\dot{e}_{0S}}\right)^n}{1 + \left(\frac{\dot{e}_i}{\dot{e}_{0S}}\right)^n},$$

where Y_d is the effective yield strength at certain time; Y_S is the stationary yield strength, which takes place in this state *(P, T)* of substance at $\dot{e}_i = 0$; $\dot{e}_i$ is the strain rate; t is the time of relaxation of elastic stress; $\left(\bar{T} = T/T_m\right)$; e_i^P is the plastic strain; G_0 is the shear modulus under normal state conditions; Y_0, a_0, k, e_1 are const. Table 1 presents constant coefficients of equations of the phenomenological elastic-plastic relaxation model.

The spherical component of stress tensor is presented as equation of state in the Mie-Grueneisen form.

$$P = \frac{\rho_{0K} C_{0K}^2}{n}\left(\delta^n - 1\right) + \Gamma_P \times \rho \times E_T, \qquad E = \frac{C_{0K}^2}{n}\left[\frac{\delta^{n-1}}{n-1} + \frac{1}{\delta} - \frac{n}{n-1}\right] + E_T, \; \Gamma_P = \Gamma_\infty + \frac{\Gamma_0 - \Gamma_\infty}{\delta^M},$$

where ρ_{0K}, C_{0K} are the density and volume sound velocity at T=0 K; Γ_P is the Grueneisen coefficient; ρ is the substance density; $\delta=\rho/\rho_{0K}$ is the relative compression; E, E_T are the internal and thermal energies respectively ($E_T = C_V \times T$); Γ_0, Γ_∞ are the Grueneisen coefficients (Γ_P) at $\rho=\rho_{0K}$ and $\delta \to \infty$, respectively,

$\frac{\rho_{0K} C_{0K}^2}{n}(\delta^n - 1) = P_e$ is the elastic component of pressure.

Dependence of melting temperature on density is determined basing on the Lindemann's equation:

$$\frac{d \ln T_m}{d \ln \rho_m} = 2\left(\Gamma_p - \frac{1}{3}\right).$$

Table 2 presents constant coefficients of the equation of state. The shear modulus, which determines the deviatoric component of stress tensor in elastic area of strain, can be calculated by the formula:

$$G = \frac{3}{2} \times \frac{1 - 2\nu}{1+\nu} \times \rho \times C_K^2, \qquad C_K^2 = \left(\frac{\partial P}{\partial \rho}\right)_S,$$

where ν is the Poisson's ratio; C_K is the current volume sound velocity. The dependence of the Poisson's ratio ν is function of temperature $\nu(T/T_m)$. It is presented as

$$\nu = \nu_0 \times \left[1 + b_0 \times \bar{T}^K\right], \ \bar{T} < 1 \qquad \nu = 0.5, \ \bar{T} \geq 1,$$

where ν_0, b_0, K=const. Table 3 includes parameters of the equation for Poisson's ratio of all considered metals.

Metal	Y_0, GPa	a_0	k	ε_1	G_0, GPa	t_0, ns	$\dot{\varepsilon}_{i0}$, $1/s$	j_{max}	$\dot{\varepsilon}_{0S}$, $1/s$	n
Al AMg6	0.20	1.4	5.0	0.1	26	2.28	1.0	0	-	0
Be	0.36	3.3	1.0	0.143	160	5	1.0	156	2×10^3	1
Cu	0.055	7.5	1.0	0.31	40	25	0.25	100	10^6	1
Ta	0.44	1.4	1.0	0.5	70	5.0	0.5	0	-	0
DU	0.4	3.1	1.0	0.285	85	5	70	250	10^6	2

Table 1

Metal	ρ_{0K}, g/cm³	ρ_0, g/cm³	ρ_{m0}, g/cm³	C_{0K}, m/s	n	Γ_∞	Γ_0	M	c_V, J/gK	T_{m0}, K
Al AMg6	2.69	2.64	2.524	5.55	3.2	0.67	2.14	1.2	0.9	933
Be	1.8668	1.85	1.707	8.03	2.89	0.50	1.25	1.0	2.7	1557
Cu	9.054	8.93	8.384	3.95	4.3	0.67	1.96	1.2	0.391	1356
Ta	16.76	16.659	15.478	3.30	3.4	0.67	1.70	1.0	0.14	3290
DU	18.9	18.7	17.76	2.62	4.6	0.70	2.3	3.4	0.105	1404

Table 2

Metal	ν_0	b_0	K
Al AMg6	0.322	0.55	3.0
Be	0.03	15.63	3.0
Cu	0.347	0.44	1.0
Ta	0.348	0.434	3.0
DU	0.168	1.976	1.0

Table 3