# Multi-Material Pressure Relaxation Methods for Lagrangian Hydrodynamics

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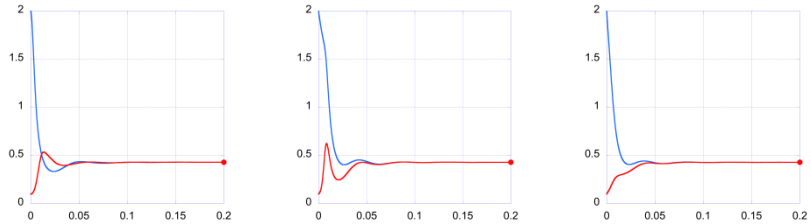
## Introduction

In Arbitrary Lagrangian-Eulerian (ALE) methods for hydrodynamics with several materials, multiple-material Lagrangian cells invariably arise when the flow field is remapped onto a new mesh. One must close the system of equations for a multi-material cells in Lagrangian hydrodynamics. Any closure of these equations forms an model, either explicit or implicit, for the sub-scale dynamics. In this presentation, we discuss multi-material closure models under the assumption of a single velocity in 1D.

**Background** In this presentation, we focus on careful comparison of various models developed by the authors. Bondarenko, Delov, Goncharov, Sadchikov, and Yanilkin [1,2,4,5] developed several models; many of these approaches use approximate sub-scale dynamics based on a linearized Riemann problem to initialize volume fraction changes, which are modified, via the materials' pressures or specific internal energies, to drive the mixed cell toward pressure equilibrium. Kamm and Shashkov [6] based their sub-cell model on that of Després and Lagoutière [3], but break the assumption of instantaneous pressure equilibration with a physics-inspired, geometry-based pressure relaxation model using a local, exact Riemann problem. Motivated by the work of Delov and Sadchikov [2] and Goncharov and Yanilkin [3], Kamm et al. [7] used the individual materials' energy equations, which are modified by an energy exchange term. Further developing this approach, Kamm et al. [8] chose a specific energy exchange term, which guarantees the energy balance of the problem.

**Discussion** We consider different underlying hydrodynamics integration schemes in addition to the different closure models; in our presentation, we discuss the details of these algorithms. We examine these methods on two idealized, 1-D test problems. In these tests, we compare results of a pure, two-material approach (i.e., where each computational cell has only one material) with that of a simulation in which one cell is mixed, i.e., contains both materials, together with a pressure-equilibration model. We consider the well-known modified Sod shock tube problem on $0 \le x \le 1$, with the initial interface at $x = 0.5$ and a final time $t = 0.2$ (all quantities are non-dimensional).

Figure 1 shows that the pressure equilibrates in each method and relaxes to the correct solution. The actual time-history of the pressures, however, differs according to the closure model. The pure-material results of the different underlying algorithms (shown in the first column) are all similar, with exchange of pressure extrema between the materials. The pressure relaxation model results, in the second and third columns, exhibit some notable differences. In our presentation, we discuss these differences, together with the time-histories of the density and specific internal energy and the differences among the methods seen in those quantities. Also, we describe results of a 1D test problem for a water-air shock tube.



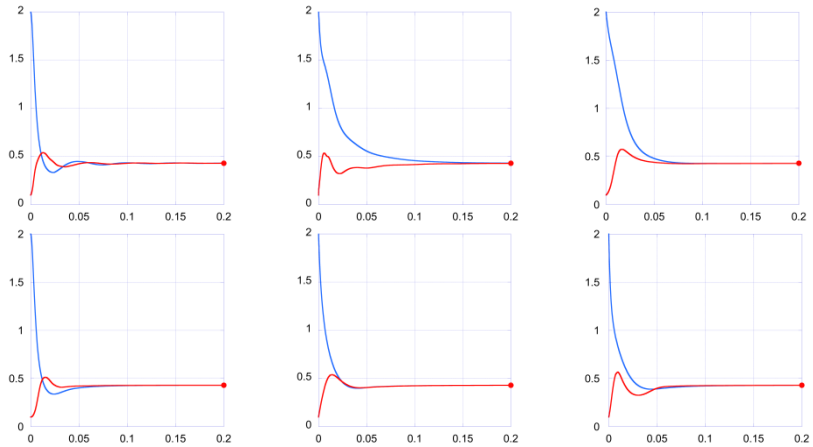


Figure 1: Plots of pressure versus time in the mixed cell for modified Sod shock tube problem. Entry (1,1)–Algorithm I, pure-material; (2,1)–Algorithm II, pure-material; (3,1)–Algorithm III, pure-material; (1,2)–Algorithm I, $\Delta P$; (1,3)–Algorithm I, $\Delta v$; (2,2)–Algorithm II, LRP; (2,3)–Tipton; (3,2)–Kamm-Shashkov-Rider; (3,3)–Kamm-Shashkov. The blue line is the left material (material 1), the red line is the right material (material 2), and the bullets represent the exact solution at the final time.

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