Patch-Based Adaptive Mesh Refinement for Hydrodynamics

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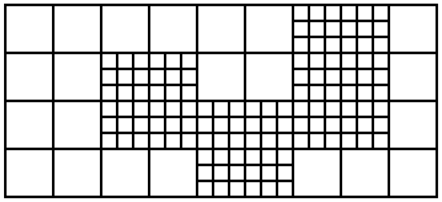
Overview

- "AMR 101" for gas dynamics
- Overview of a single material algorithm on a structured Eulerian grid
- Issues for multimaterial cells
- Assorted results

Berger-Oliger-Colella style adaptive mesh refinement

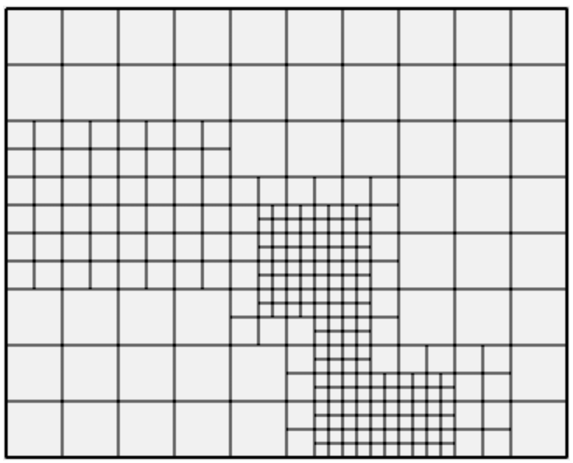
- Features of Berger-Oliger-Colella approach
  - Patch consists of high "error" zones grouped along with some (but not many) low-error zones
  - Locally refine patches of the domain in space and time
  - Each patch is a topologically rectangular structured grid
  - Patches are "properly nested"
  - Grids are dynamically created and destroyed to allow for changing features of unsteady flow
  - Patches vary in size spatially and temporally
  - Subcycling in time (recursive time step) is possible and routinely used, not necessary

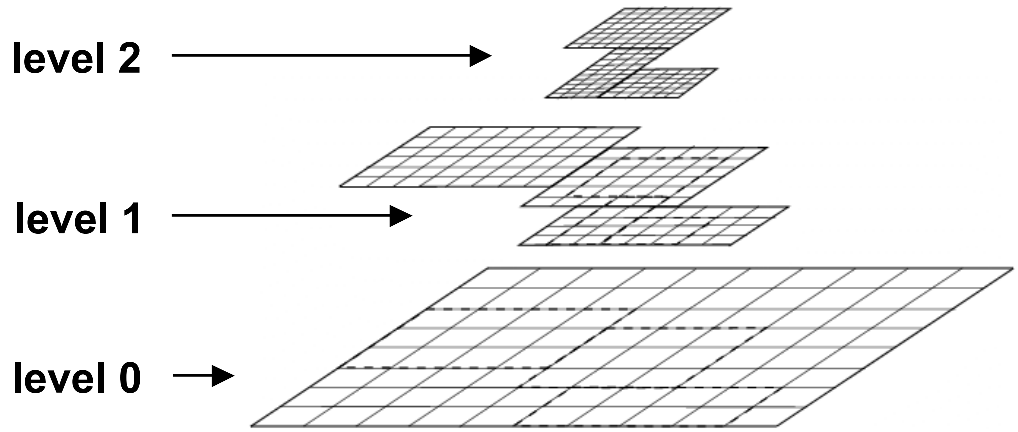


- For the purpose of this talk
  - Physically rectangular grids
  - Cell-centered variables
  - Single level time advance is explicit, direct Eulerian, discretely conservative, 2nd order accurate, structured grid
    - structured grid advance achieved through use of ghost zones
    - 2nd order accurate → linear interpolations are sufficient

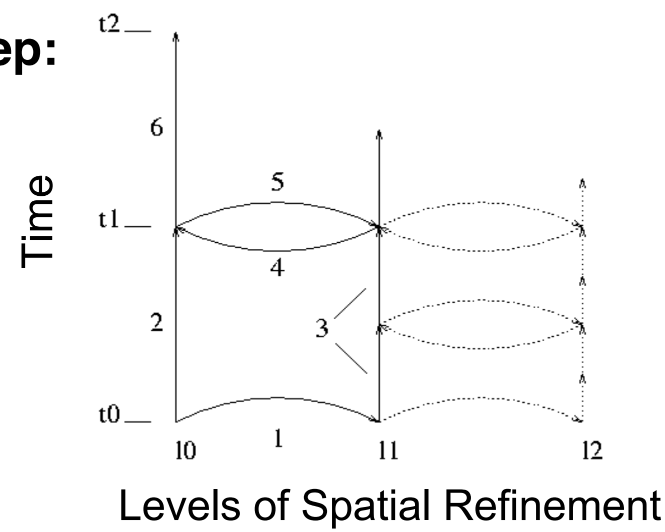
Berger-Oliger-Colella AMR uses dynamic hierarchy of meshes and recursive time step

- **Spatial refinement**





- **(Recursive) coarse level time step:**
  - **Advance coarse level (1,2)**
  - **Advance fine level (3)**
  - **Synchronize levels (4,5)**
  - **Regrid current and all finer levels**



Single material equations

- **Conservation laws for mass, momentum and energy**

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial G(U)}{\partial y}+\frac{\partial H(U)}{\partial z}=0$$

- **Equations for history dependent variables:** $$\frac{\partial \rho\phi_i}{\partial t}+\nabla\cdot(\rho\phi_i\mathbf{v})=\Phi(\rho,\varepsilon,\phi_1,\ldots,\phi_n)$$

- **Evolution equation for elastic shear deformation:**
  - **Elastically isotropic response**
  - **Invariant relative to superimposed rigid body motion**
  - **Handles large deformation correctly**
  - **Separated from volumetric response**
  - **Suitable for hyperelastic formulation based on Helmholtz free energy**

Evolution of elastic shear deformations

- **Non-conservative formulation**

$\mathbf{B}_e$ – left symmetric tensor of elastic deformations

$\mathbf{B}'_e = \det(\mathbf{B}_e)^{-1/3}\,\mathbf{B}_e$ – unimodular (pure shear) tensor

$$\boxed{\dot{\mathbf{B}}'_e = \mathbf{L}\cdot\mathbf{B}'_e + \mathbf{B}'_e\cdot\mathbf{L}^{\mathrm{T}} - 2/3\,(\mathbf{L}:\mathbf{I})\,\mathbf{B}'_e - \mathbf{A}_p}$$

$\mathbf{L} = (\nabla\otimes\mathbf{v})^{\mathrm{T}}$ – velocity gradient, $\mathbf{A}_p$ – plasticity operator

- **Helmholtz free energy**

$$\rho_{s0}\psi = \rho_{s0}\psi_1(J_e,\theta) + 1/2\,G(J_e,\theta)(\mathbf{B}'_e:\mathbf{I}-3)$$

$J_e = \det(\mathbf{B}_e)^{1/2}$, θ – temperature, G – shear modulus

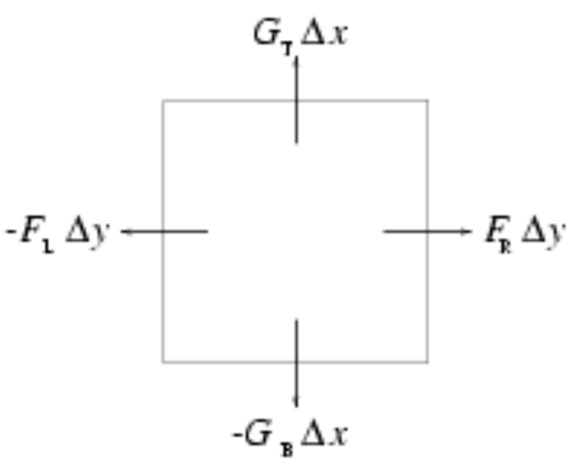
Explicit conservative scheme example

- finite difference scheme $\equiv$ approximate $\nabla \cdot \vec{\boldsymbol{F}}$ $\quad (\vec{\boldsymbol{F}} = (F, G))$
- recall:

$$\int \nabla \cdot \vec{\boldsymbol{F}} dV = \int \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}} dA$$

- use divergence theorem

$$\nabla \cdot \vec{\boldsymbol{F}} \approx \frac{1}{\Delta x \Delta y} \sum_i \vec{\boldsymbol{F}}_i \cdot \vec{\boldsymbol{n}}_i A_i, \quad i = \mathrm{L, R, T, B}$$



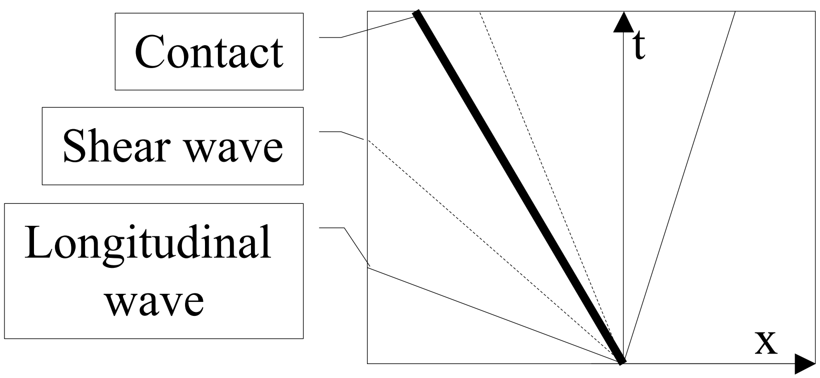
- update is $U^{n+1} = U^n - \Delta t \nabla \cdot \vec{\boldsymbol{F}}$

High order Godunov method for solids

- **Characteristic tracing**
  - **Traced variables:**

$$\begin{aligned}&\rho - \text{density},\\ &\rho\varepsilon - \text{internal energy},\\ &\mathbf{v} - \text{velocity},\\ &\mathbf{T} - \text{stress tensor},\\ &\phi_i - \text{history dependent variables}\end{aligned}$$

- **Riemann solver**
  - **acoustic approximation for shear waves**
  - **nonlinear solver for longitudinal waves**



Constitutive equation outline

- Framework of porosity and general plasticity models
- Rate-dependent formulation
- Yield function $Y = Y_0 F_1(\varepsilon_p, p) F_2(\beta, p) F_3(\rho, \varepsilon) F_4(\phi) F_5(\omega, p)$
  - plastic strain and pressure hardening
  - Löde angle dependence
  - Thermal and structural softening
- Porosity evolution
  - $p - \alpha$ model for initial porosity compaction
  - shear enhanced compaction
  - spall, directional tensile failure
  - dilatancy under positive pressure (bulking)

Volume of fluid approach

- **Thermodynamics based equations for the mixed cell update:**

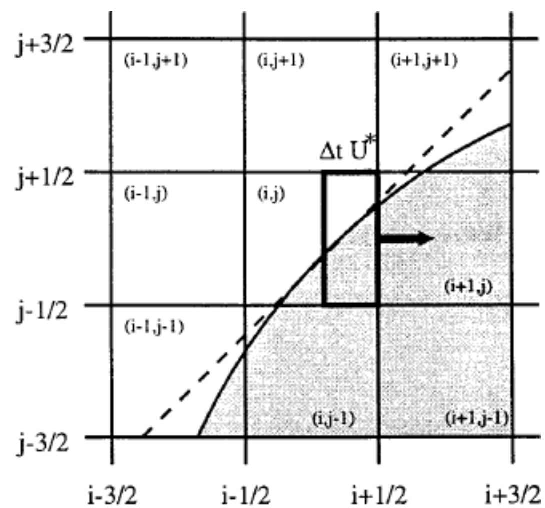
$$\frac{\partial f_\alpha}{\partial t} + \nabla \cdot (f_\alpha \mathbf{v}) = \frac{f_\alpha}{K_\alpha} K \nabla \cdot \mathbf{v}$$

$$f_\alpha - \text{volume fraction of fluid } \alpha$$

$$K_\alpha - \text{bulk modulus of fluid } \alpha$$

$$1/K = \sum f_\alpha / K_\alpha$$

- **High order interface reconstruction (preserves linear interface during translation)**



Mixed cell treatment

- **Effective cell properties:**

$$1/K = \sum f_\alpha / K_\alpha - \text{bulk modulus}$$

$$1/G = \sum f_\alpha / G_\alpha - \text{shear modulus}$$

stress tensor components:

$$T_{ii} = 1/K \sum f_\alpha T_{ii\alpha} / K_\alpha - \text{diagonal}$$

$$T_{ij} = 1/G \sum f_\alpha T_{ij\alpha} / G_\alpha - \text{non-diagonal}$$

- **Distribution of velocity gradient for $\mathbf{B}_e'$ update:**

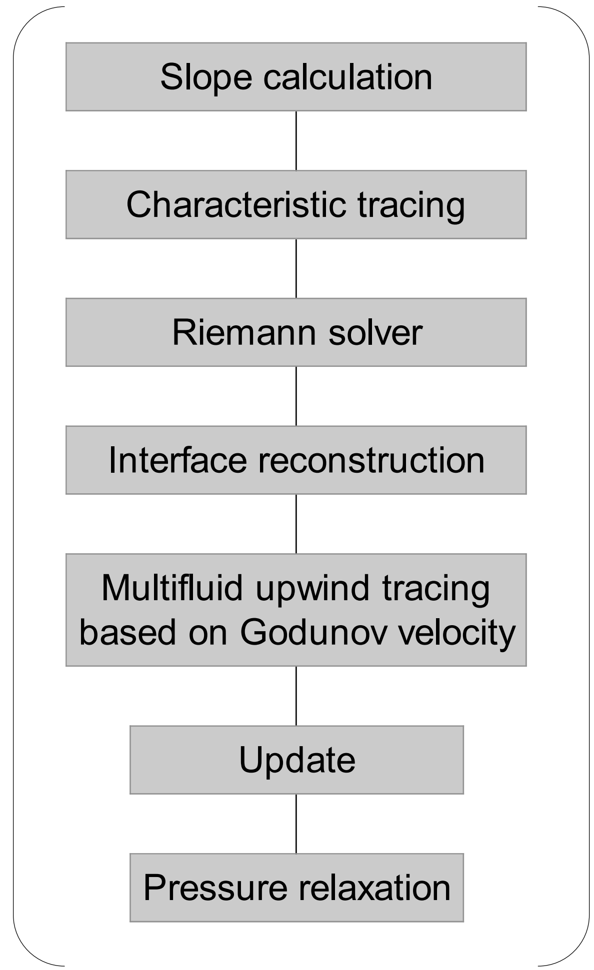
$$\mathbf{L}_\alpha = \mathbf{L} G / G_\alpha$$

Solution advance algorithm on a single grid:

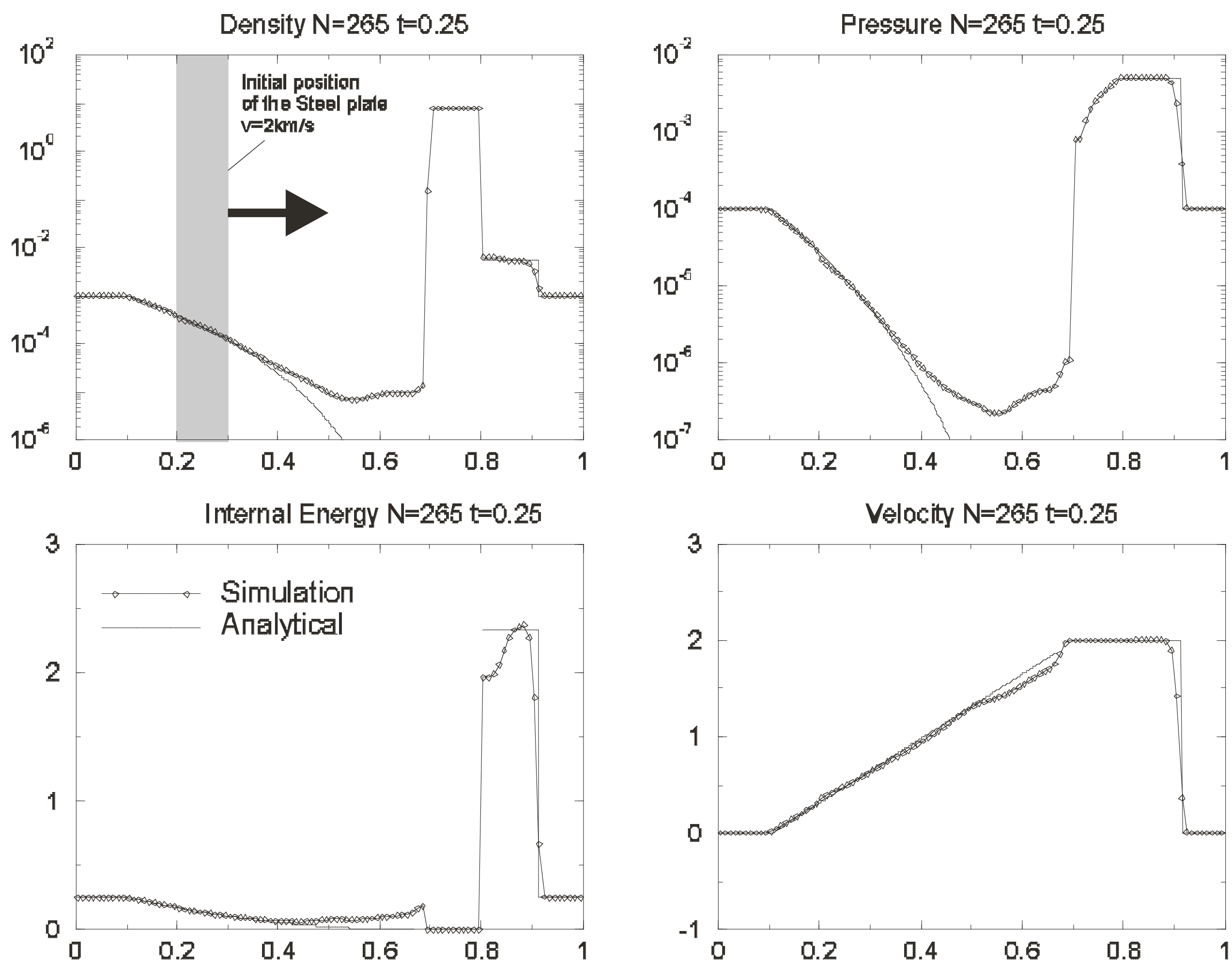
- **Operator split approach:**
  - **Physics split**
  - **Dimensional split**

$$2L_{full} = L_x L_y L_z L_{source} L_z L_y L_x L_{source}$$

- **1D sweep**

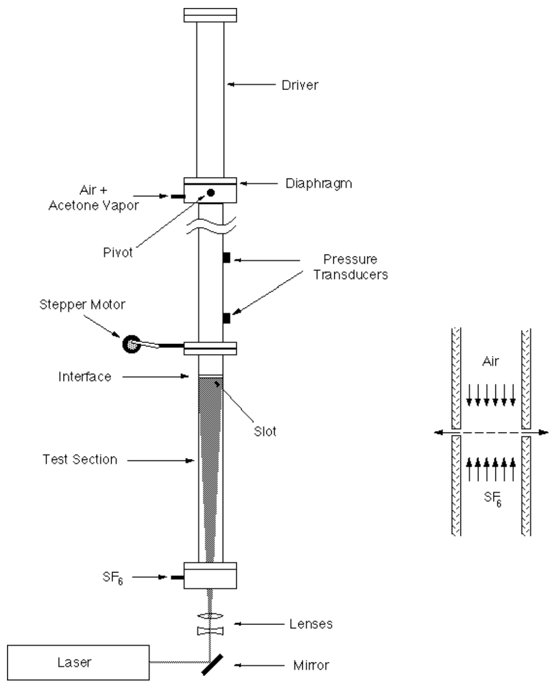


1D test: motion of a metal plate in air



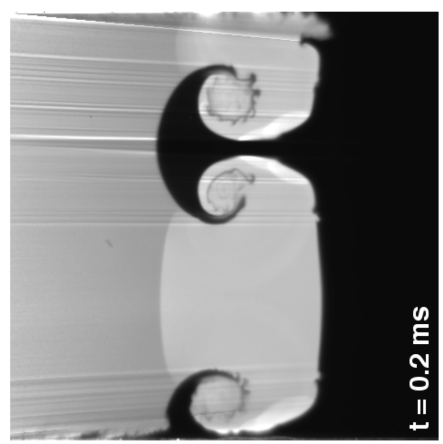
Raptor (Boxlib) computation of “re-shock” at M=1.3, Air/SF6

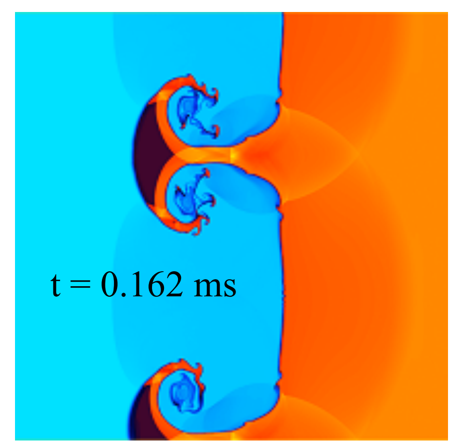
Vertical Shocktube at the University of Arizona (Prof. Jeff Jacobs)



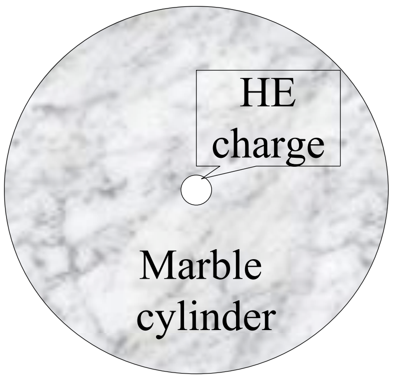
AMR is well-suited for computing reshock

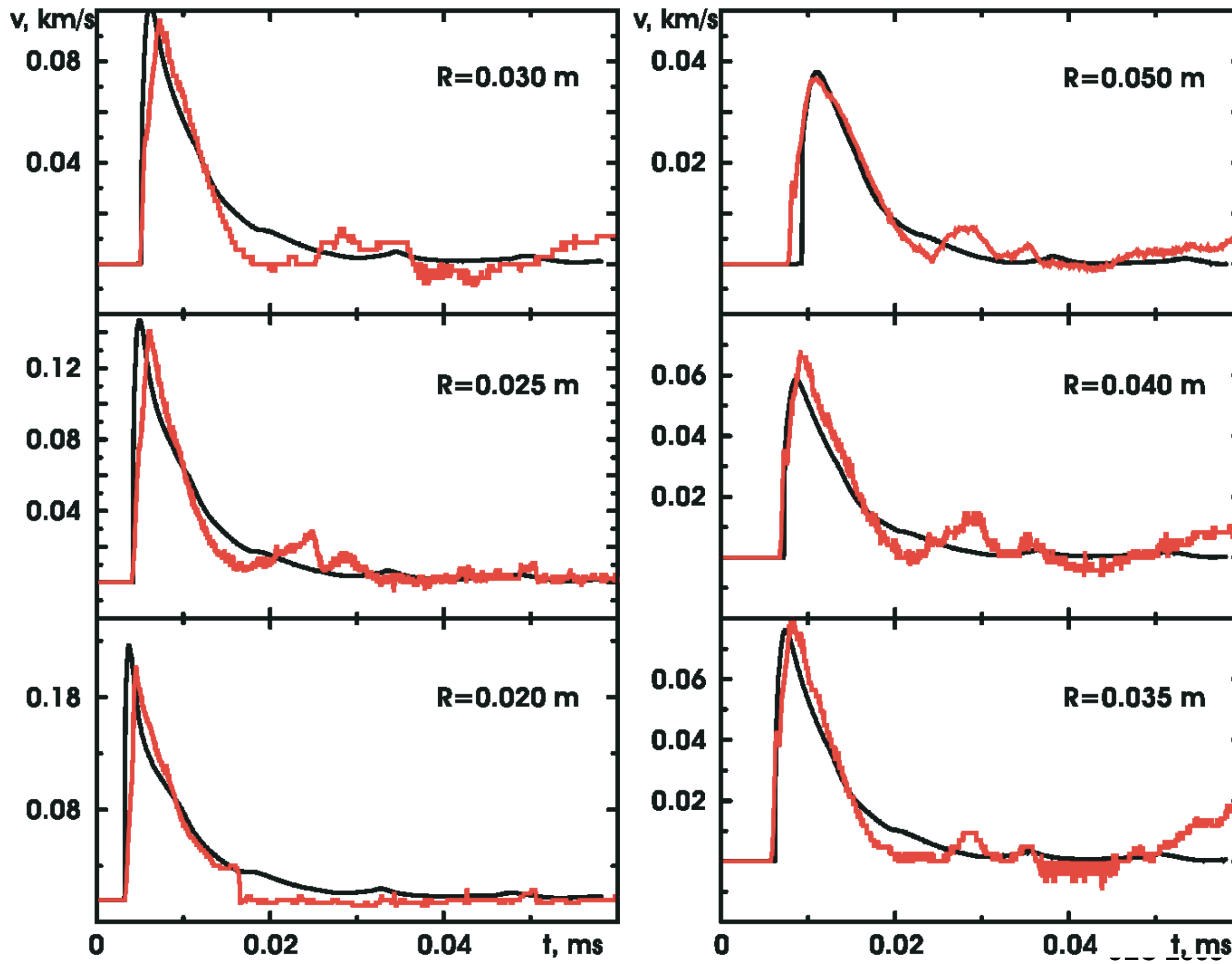
- Ignore small time offset due to experimental false bottom



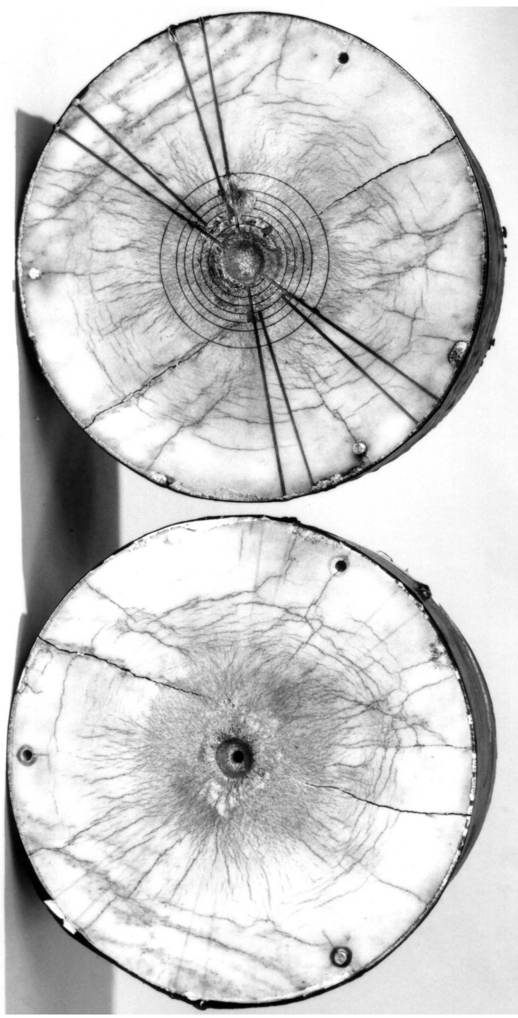


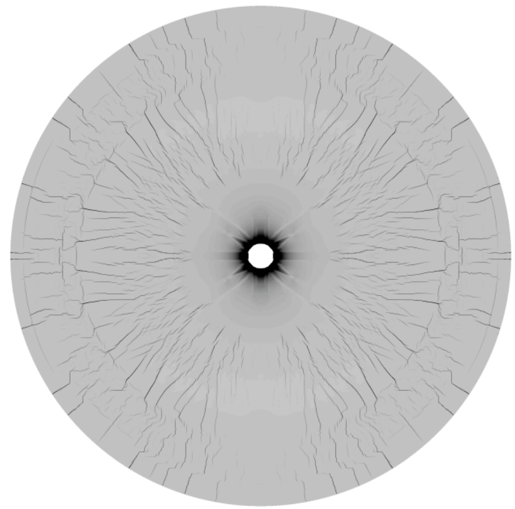
Spherical wave propagation test in marble



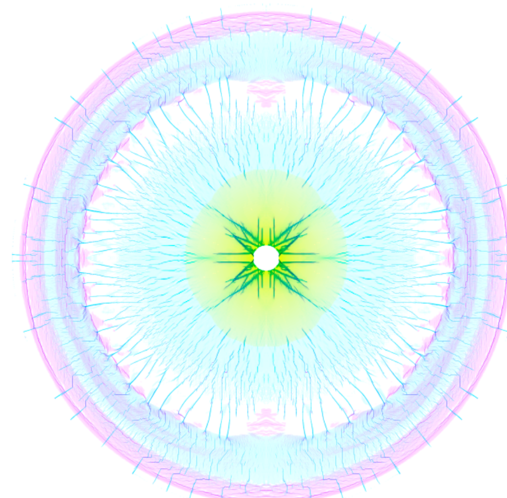


Explosion in a marble cylinder





Porosity

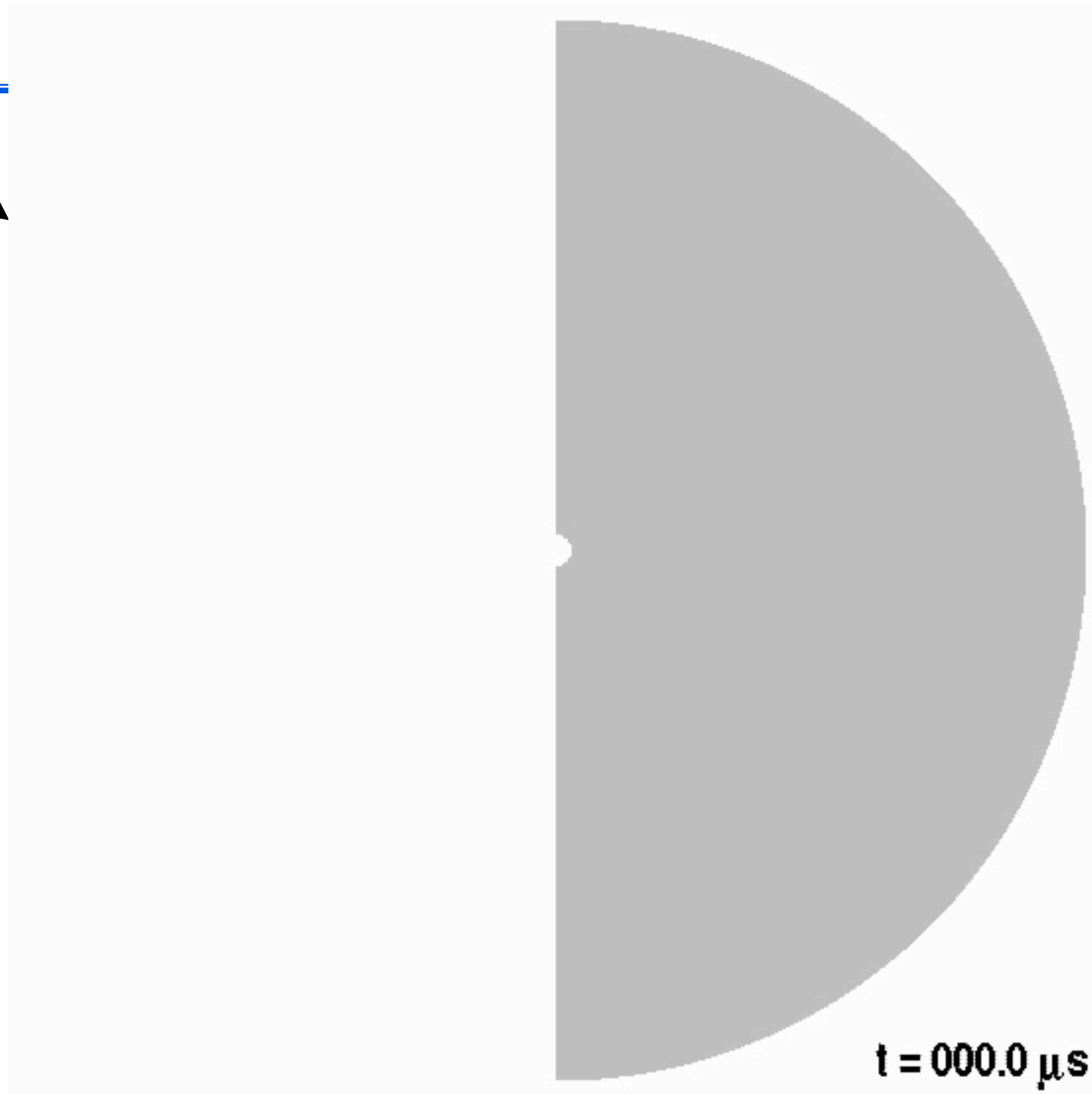


Composite

damage

Cracking damage

Void Volume Fraction

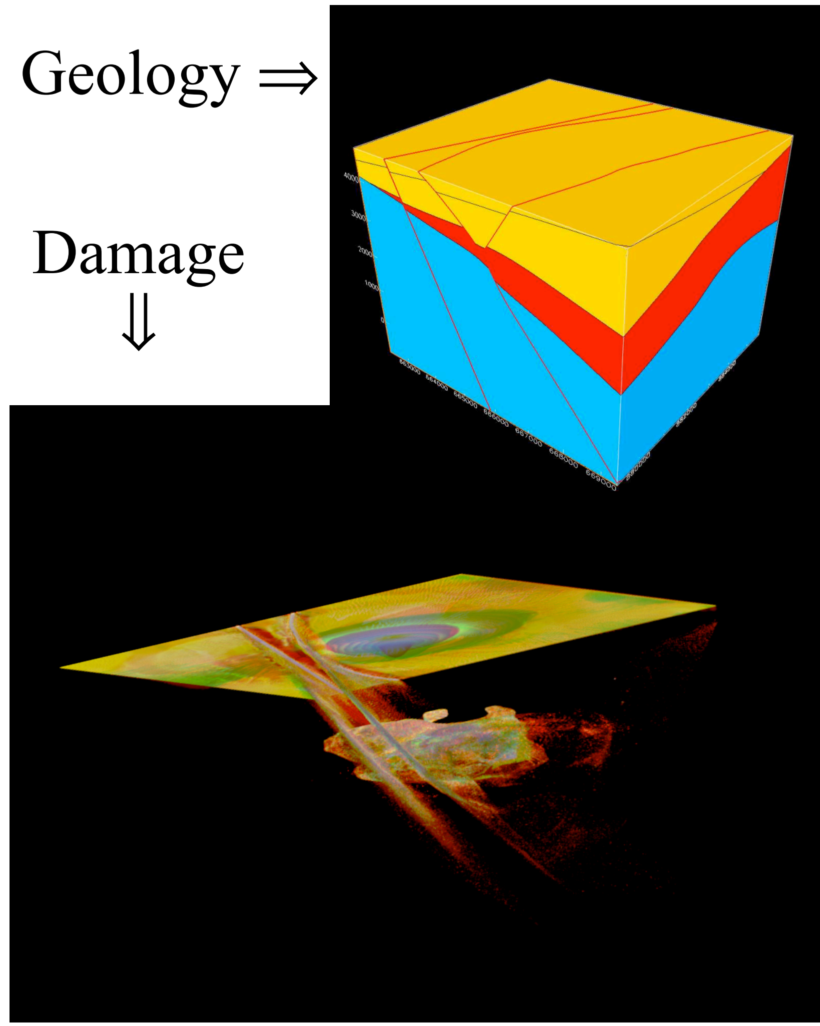


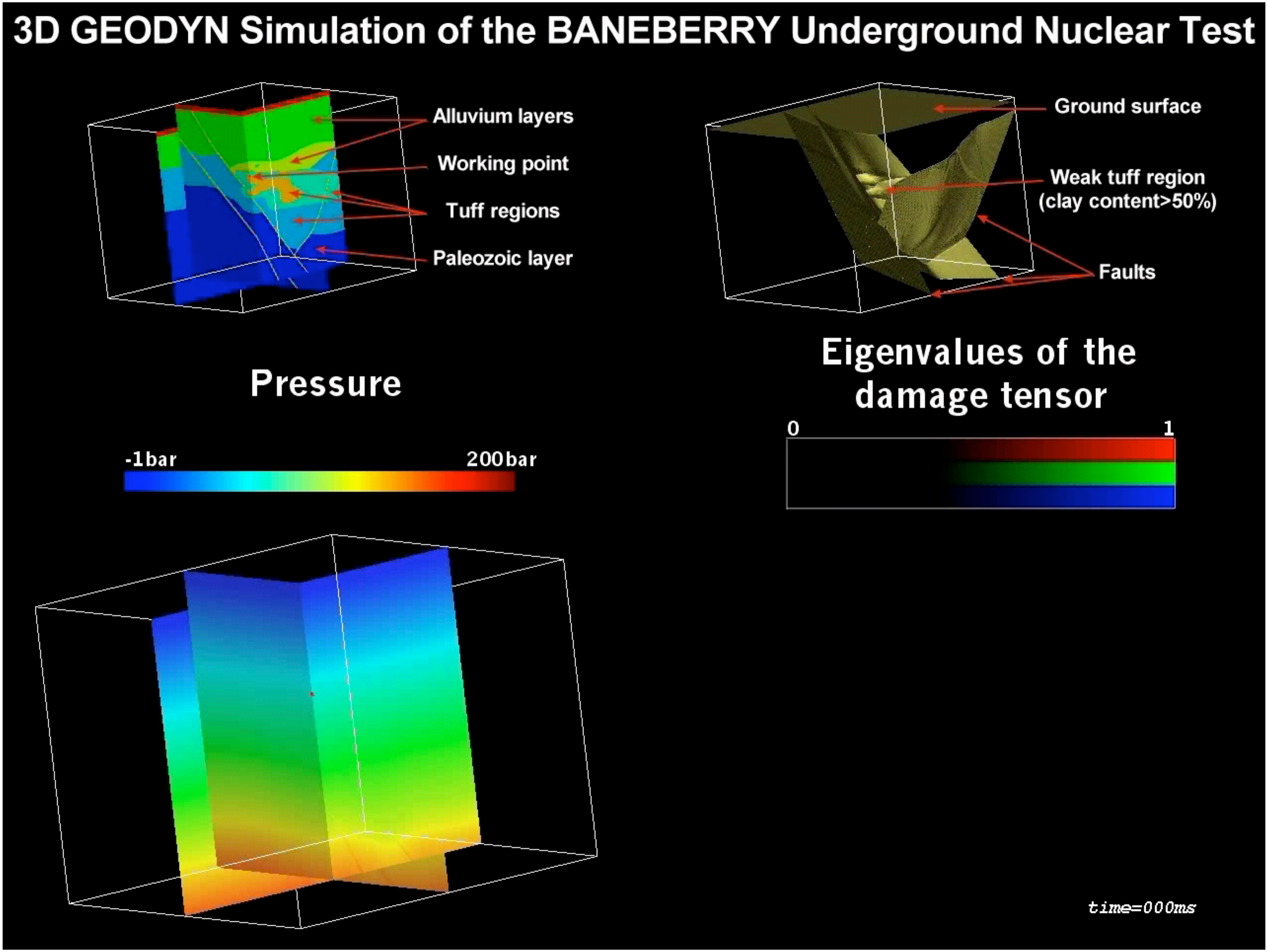
Baneberry underground nuclear test (1970)

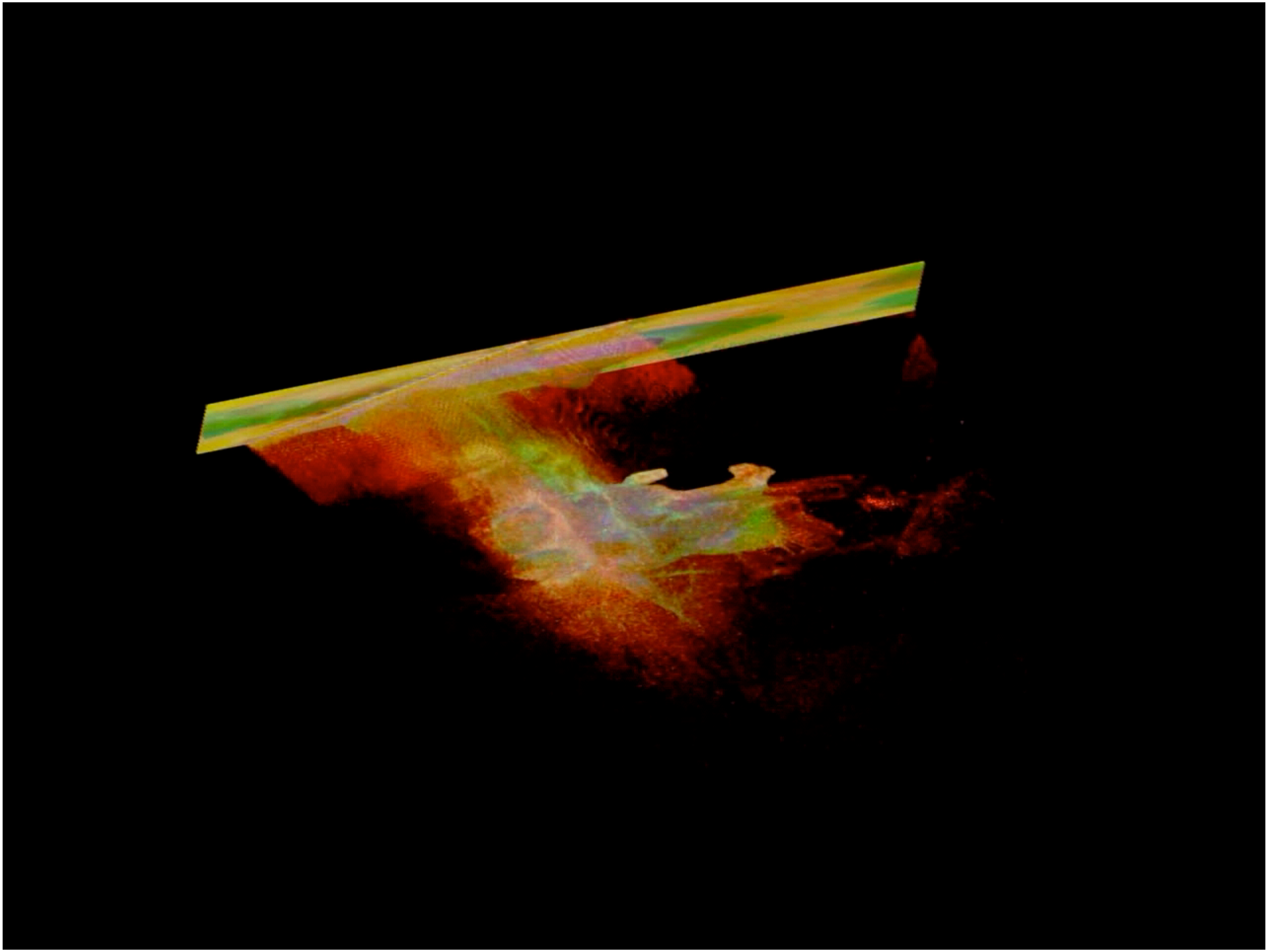
Geology ⇒

Damage

⇓




3D GEODYN Simulation of the BANEBERRY Underground Nuclear Test
Alluvium layers
Working point
Tuff regions
Paleozoic layer
Ground surface
Weak tuff region
(clay content>50%)
Faults
Pressure
-1bar
200bar
Eigenvalues of the damage tensor
0
1
time=000ms



Current work

- Special things which we need to do when material interface crosses boundary between refinement levels
- Use information from single-material cells nearby for better representation of materials in the mixed cell
- Different options for the energy update in the mixed cells
- How do we simulate gas diffusion and mixing while using tracking algorithms for solid and liquids?