Institute of Theoretical and Mathematical Physics

Russian Federal Nuclear Center -

VNIIEF

The Technique for Solving the 2D Transport Equation Using Irregular Polygonal Grids

A.N.Moskvin and V.A. Shumilin

RFNC-VNIIEF, 607190, Sarov, Nizhni Novgorod region

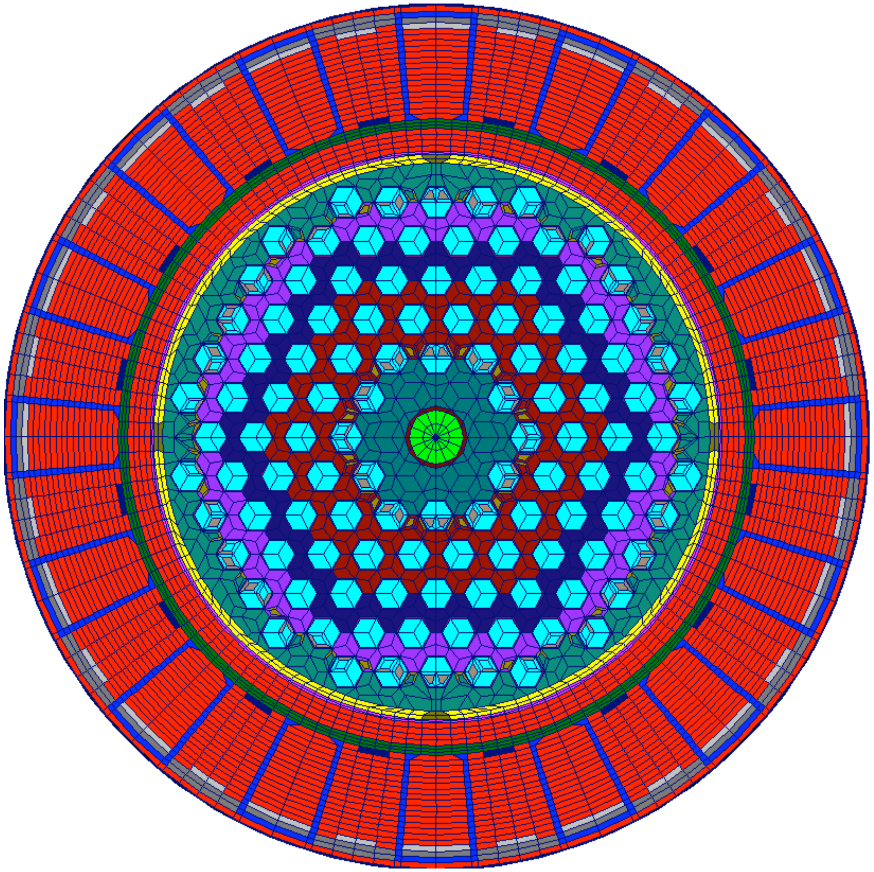
Joint Russian-American Five-Laboratory Conference on Computation Mathematics|Physics

19-23 June 2005, Crowne Plaza, Vienna Austria

Abstract

A conservative finite difference scheme for polygonal spatial grids has been constructed to solve the 2D transport equation. The difference scheme is constructed in two phases. During the first phase temporary values of the unknown function in a computational cell are to be found by approximately solving the transport equation along characteristics. During the second phase the balance equation is used to find the correction factor and all the obtained values of the unknown function are multiplied by this factor. There have been developed cost-efficient algorithms implementing the sweep method using spatial grids which components are polygons of arbitrary shapes. The technique serviceability is demonstrated using the results of numerical simulations.

Abstract



Example of the calculated system
(Container for keeping perfected element TVS)

Statement of Problem

Consider a time-dependent 2D kinetic equation of particle transport in axial-symmetry geometry in multigroup approximation. In cylindrical system of coordinates and with the divergent form of writing the equation looks like:

$$\frac{\partial}{dt}\left(\frac{N_i}{\upsilon_i}\right) + L\,N_i + \alpha\,N_i = F_i \qquad (1)$$

$$L\,N_i(z,r,\mu,\varphi) = \frac{\partial}{r\cdot\partial r}\left(r\cdot\sqrt{1-\mu^2}\cdot\cos\varphi\cdot N_i\right) + \frac{\partial}{\partial z}(\mu\cdot N) - \frac{\partial}{\partial\varphi}\left(\frac{\sqrt{1-\mu^2}}{r}\cdot\sin\varphi\cdot N_i\right)$$

$$F_i(z,r) = \frac{1}{2\pi}\left(\sum_{j=1}^{i1}\beta_{i\,j}\cdot n_j^{(o)} + Q_i\right) \qquad n_j^{(o)} = \int_{-1}^{1} d\mu \int_{0}^{\pi} N_i\, d\varphi$$

Approximation in Angular Variables and Time

The time derivative in (1) is approximated in the following manner:

$$\frac{\partial}{\partial t}\left(\frac{N}{\upsilon}\right) = \frac{\hat{O}^{\,n+1} - \hat{O}^{\,n}}{\Delta t^{\,n+1}} \qquad (2)$$

It is assumed, when approximating in time, that all quantities depending on ***r*** and ***z*** are specified at time:

$$t^{\,n+\gamma} = (1-\gamma)\cdot t^{\,n} + \gamma \cdot t^{\,n+1}\,, \quad (0.5 \le \gamma \le 1)$$

The additional time correlation is inequality of the form:

$$\frac{N^{n+\gamma}}{\upsilon} = \gamma \cdot \hat{O}^{\,n+1} + (1-\gamma)\cdot \hat{O}^{\,n} \qquad (3)$$

For variable φ, the following additional correlation is used:

$$N_{m,\,q-1} = \eta\, N_{m,q} + (1-\eta)\, N_{m,q-1},\ (0.5 \le \eta \le 1) \qquad (4)$$

Approximation in Angular Variables and Time

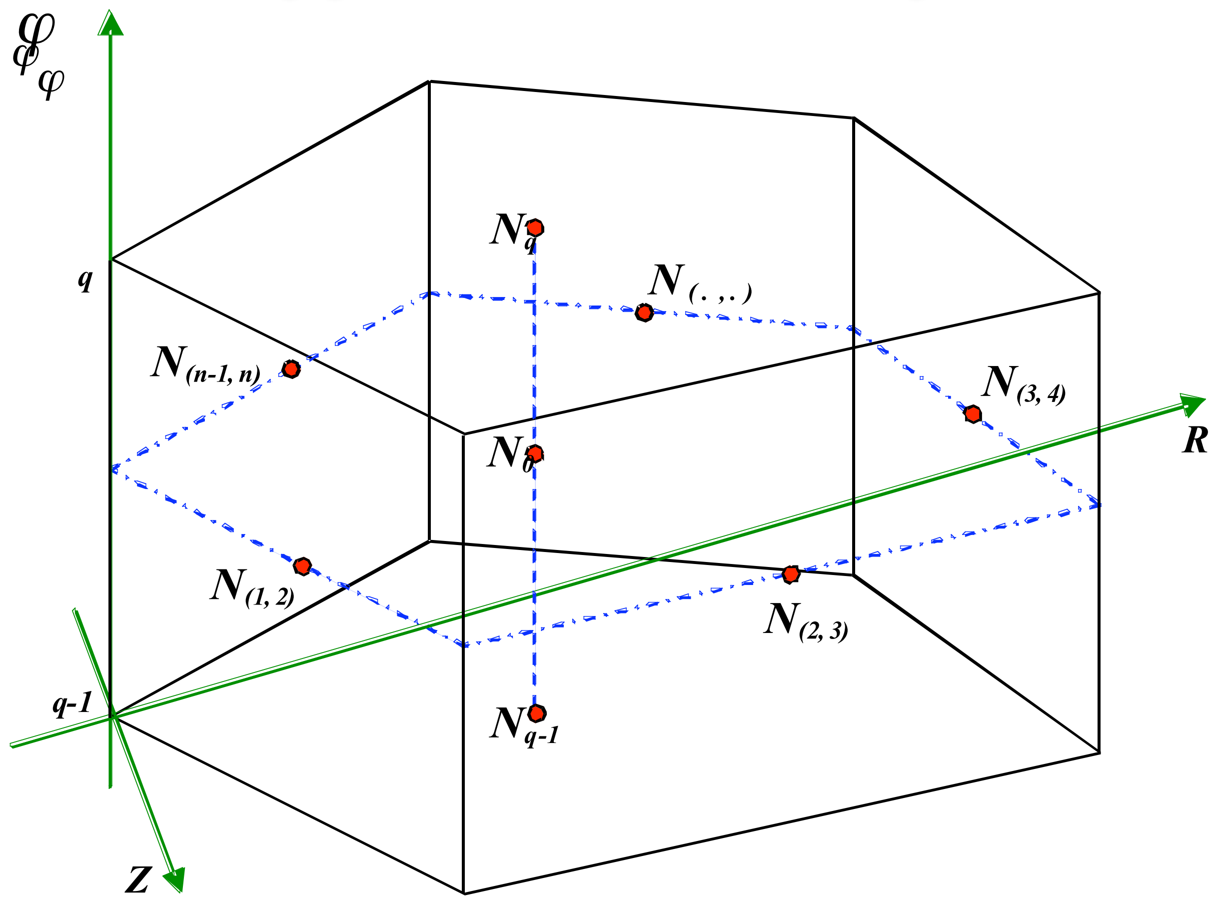
With regard to (2)-(4), Eq.1 within the ranges $\left(\varphi_{q-1},\ \varphi_q\right)$ and with the specified value of μ_m is approximated by the following difference equation:

$$
\begin{aligned}
&\frac{\partial}{r\cdot\partial r}\left(r\cdot\sqrt{1-\mu_m^2}\cdot\cos\hat{\varphi}_q\cdot N^{n+\gamma}_{m,q-\frac{1}{2}}\right)+\frac{\partial}{\partial z}\left(\mu_m\cdot N^{n+\gamma}_{m,q-\frac{1}{2}}\right)-\\
&-\frac{1}{r}\cdot\sqrt{1-\mu_m^2}\cdot\sin\varphi_q\cdot\frac{N^{n+\gamma}_{m,q}}{\Delta\varphi_q}+\frac{1}{r}\cdot\sqrt{1-\mu_m^2}\cdot\sin\varphi_{q-1}\cdot\frac{N^{n+\gamma}_{m,q-1}}{\Delta\varphi_q}+\\
&+\left(\alpha+\frac{1}{\upsilon\gamma\,\Delta t}\right)\cdot\Delta\mathrm{V}\cdot N^{n+\gamma}_{m,q-\frac{1}{2}}=\Delta\mathrm{V}\left(F^{\,n+\gamma}+\frac{\hat{O}^n\left(r,z,\mu,\varphi\right)}{\gamma\,\Delta t}\right),
\end{aligned}
\qquad (5)
$$

$$
m=1,2,\dots,\hat{m},\quad q=1,2,\dots,\hat{q}_m\ ,
$$

where: $\Delta\varphi_q=\varphi_{q-1}-\varphi_q\ ,\quad \cos\hat{\varphi}_q=\dfrac{1}{\varphi_q-\varphi_{q-1}}\cdot\displaystyle\int_{\varphi_{q-1}}^{\varphi_q}\cos\varphi\ d\varphi=\dfrac{\sin\varphi_q-\sin\varphi_{q-1}}{\varphi_q-\varphi_{q-1}}$

Approximation is Space



Example of a computational cell with the grid values of function $N(t, r, z, \mu, \varphi)$

Approximation is Space

Use Gauss-Ostrogradsky formulation for integrtion on edges and then the “mean value” theorem and obtain:

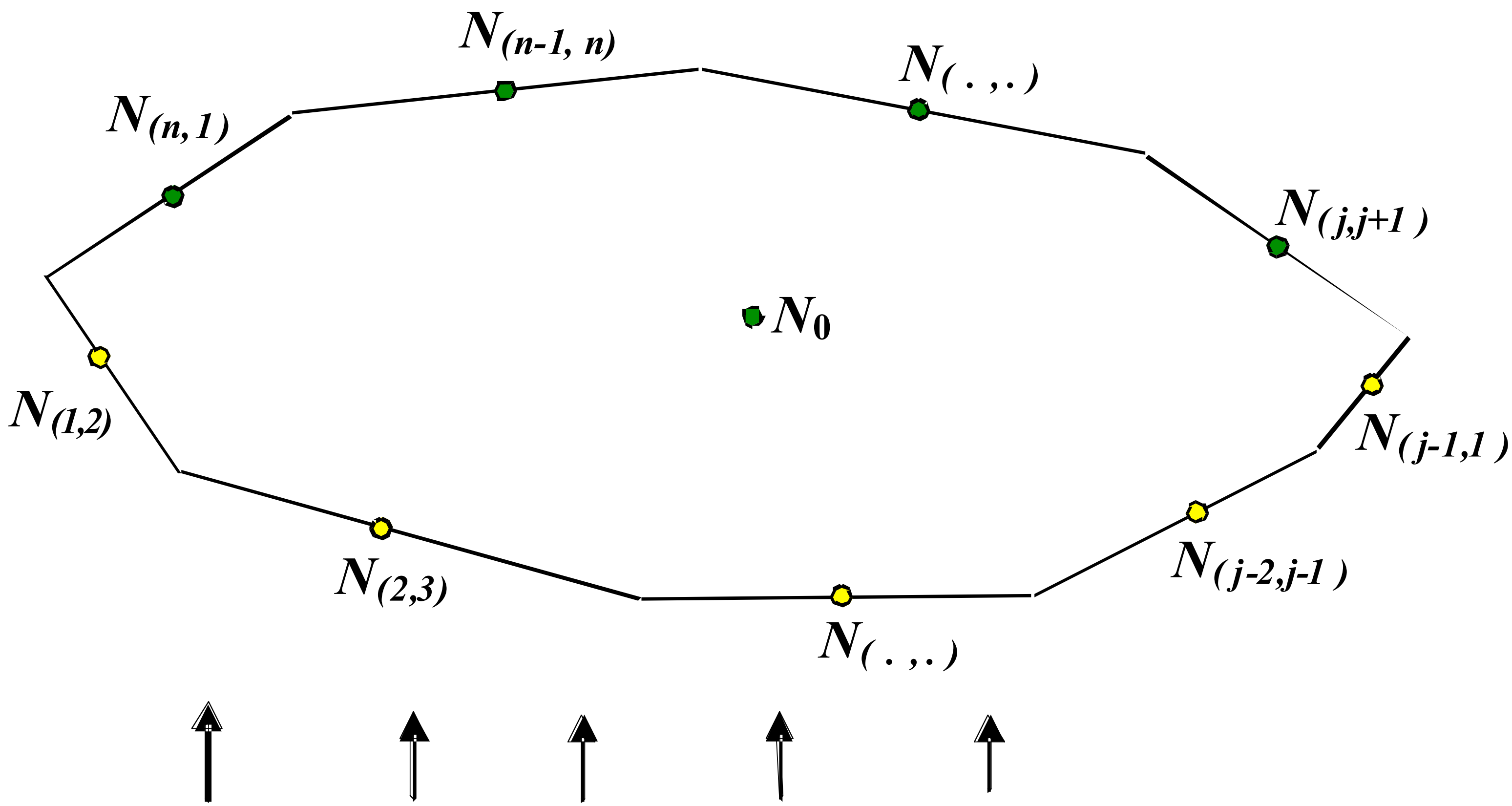
$$\sum_{k=1}^{\widetilde{k}} \chi_k \cdot N_{(k,k+1)} + N_q \cdot \chi_q - N_{q-1} \cdot \chi_{q-1} + \left(\alpha + \frac{1}{\upsilon \gamma \, \Delta t}\right) \cdot \Delta V \cdot N_0 = \Delta V \cdot \left(F + \frac{N^n}{\upsilon \gamma \, \Delta t}\right) \qquad (6)$$

$$\chi'_k = \mu_m \left(r_{k+1} - r_k\right) - \sqrt{1 - \mu_m^2} \cos \widehat{\varphi}_q \left(z_{k+1} - z_k\right) \qquad (7)$$

$$\chi_k = \frac{r_k + r_{k+1}}{2} \cdot \chi'_k, \quad r_{\widetilde{k}+1} \equiv r_1 \,, \quad z_{\widetilde{k}+1} \equiv z_1$$

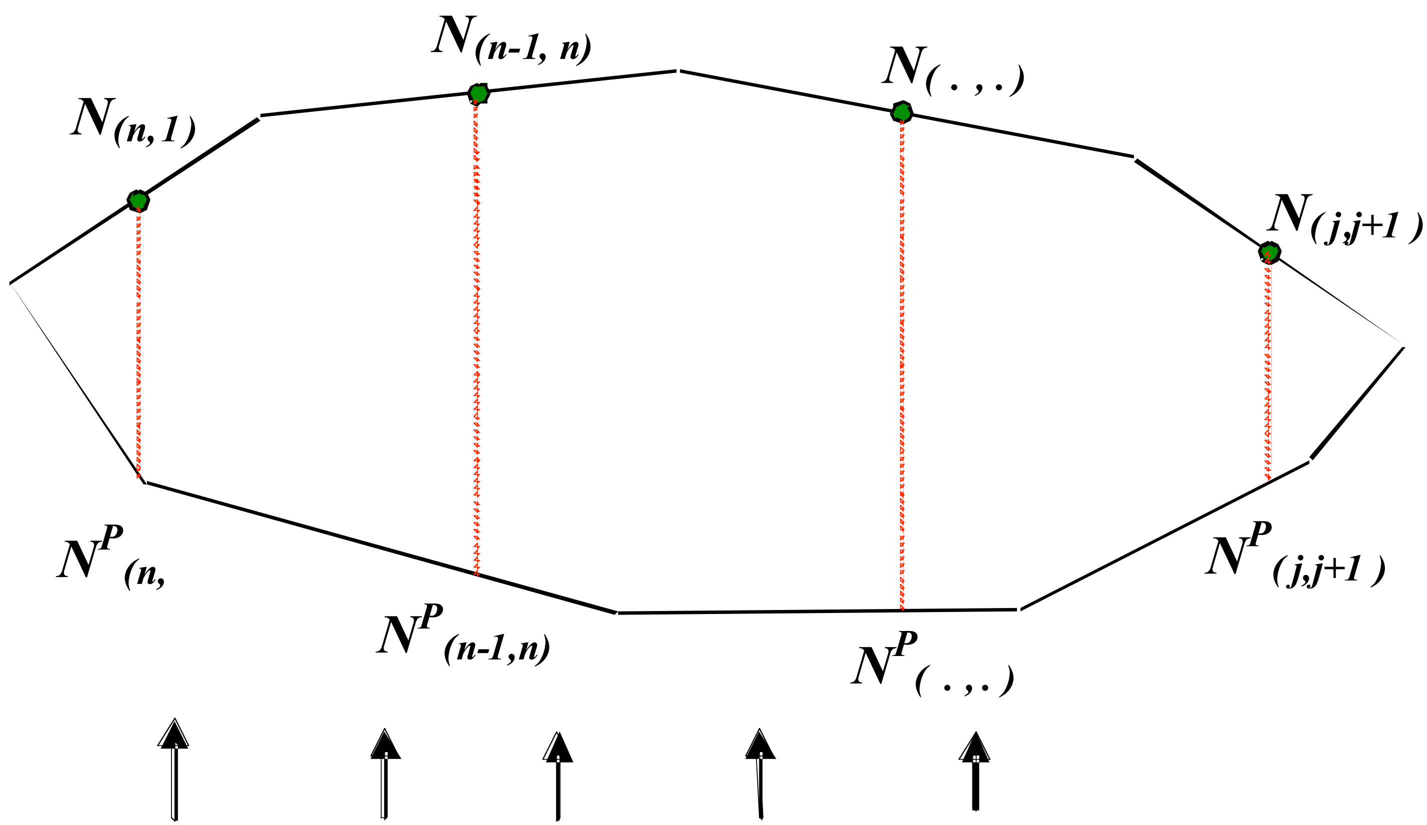
$$\chi_{q-1} = \frac{\sqrt{1 - \mu_m^2} \cdot \sin \varphi_{q-1} \cdot dS}{\varphi_{q-1} - \varphi_q} \qquad \chi_q = \frac{\sqrt{1 - \mu_m^2} \cdot \sin \varphi_q \cdot dS}{\varphi_{q-1} - \varphi_q}$$

Approximation is Space



$$N_0 = \frac{1}{n} \cdot \sum_{i=1}^{n} N_{(i,i+1)} \qquad (8)$$

Approximation is Space



$$N_{(k,k+1)} = \left(1 + d \cdot \ell_{(k,k+1)}\right) N^{P}_{(k,k+1)} \qquad \left(j \le k \le n\right) \qquad (9)$$

Approximation is Space

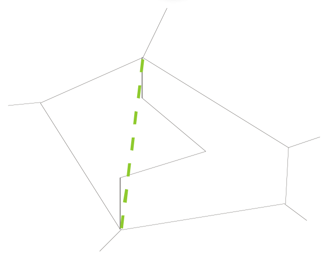
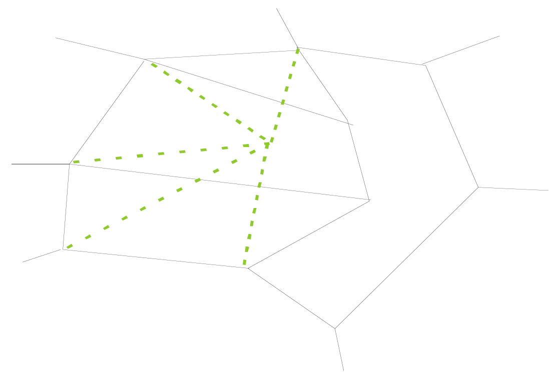
Substitute Eq.8 and Eq.9 into Eq.6 and obtain the equation for calculation of parameter d for the cell of interest:

$$
\begin{aligned}
& d \cdot \left\{ \sum_{\chi>0} \ell_{(k,k+1)} \cdot N^{P}{}_{(k,k+1)} \cdot \chi_k + \frac{1}{n} \left(\sum_{\chi>0} \ell_{(k,k+1)} \cdot N^{P}{}_{(k,k+1)} \right) \cdot \left(\frac{\chi_q}{\eta} + \left(\alpha + \frac{1}{\upsilon \gamma \Delta t} \right) \cdot \Delta V \right) \right\} = \\
& = \Delta V \cdot \left(F + \frac{N^n}{\upsilon \gamma \Delta t} \right) - \sum_{\chi<0} N_{(k,k+1)} \cdot \chi_k - \sum_{\chi>0} N^{P}{}_{(k,k+1)} \cdot \chi_k - \left(\frac{\chi_q}{\eta} + \left(\alpha + \frac{1}{\upsilon \gamma \Delta t} \right) \cdot \Delta V \right) \cdot \\
& \cdot \left(\frac{1}{n} \sum_{\chi<0} N_{(k,k+1)} + \frac{1}{n} \sum_{\chi>0} N^{p}{}_{(k,k+1)} \right) - N_{q-1} \left(\frac{\chi_q \cdot (1-\eta)}{\eta} - \chi_{q-1} \right)
\end{aligned}
\qquad (10)
$$

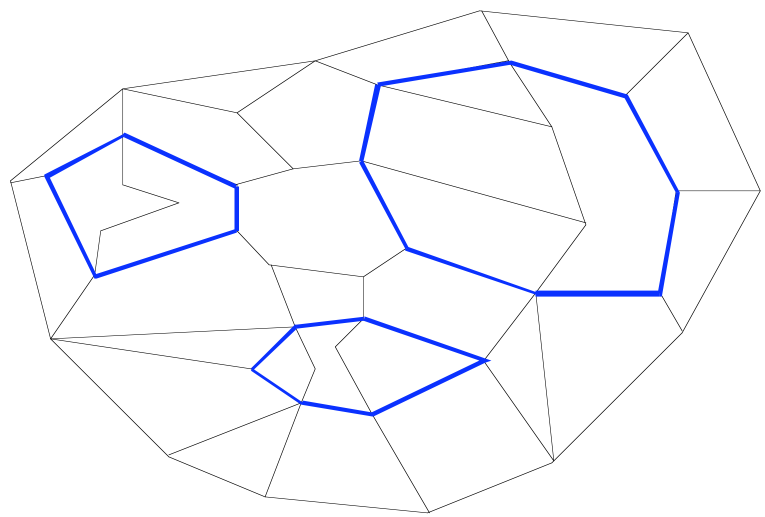
$\sum\limits_{\chi>0}$ - is the sum over all non-illuminated sides;

$\sum\limits_{\chi<0}$ - is the sum over all illuminated sides.

Of regulating polygons in a region

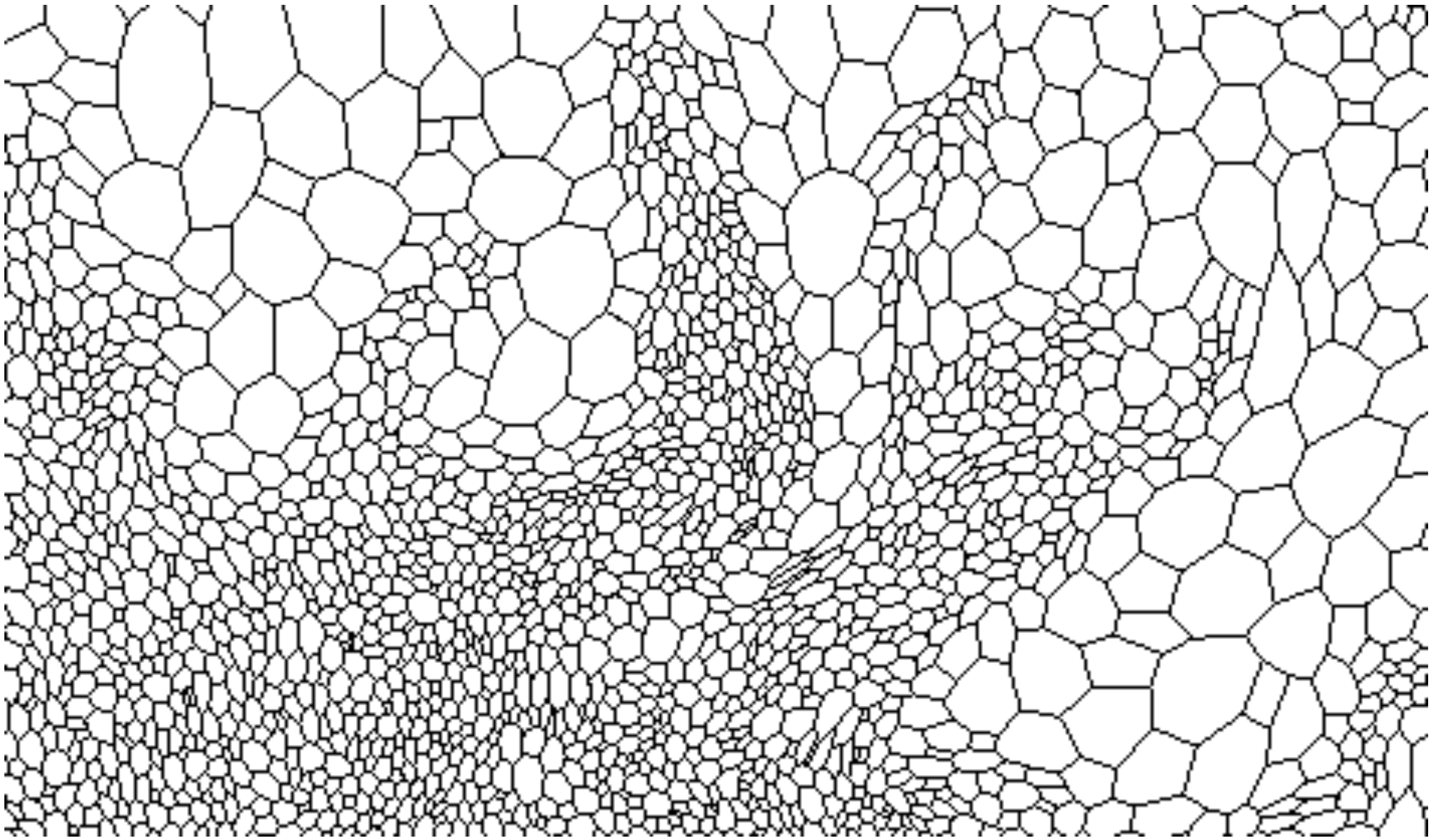



Example of algorithms avoiding non-convex polygons



Example of identification of local sub-regions with non-convex cells

Of regulating polygons in a region

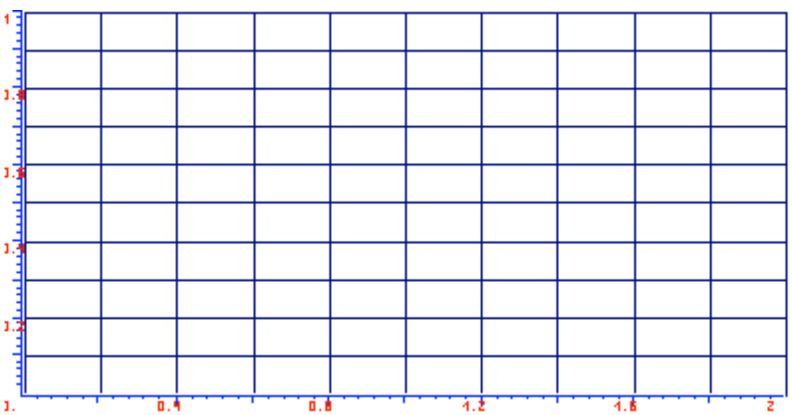


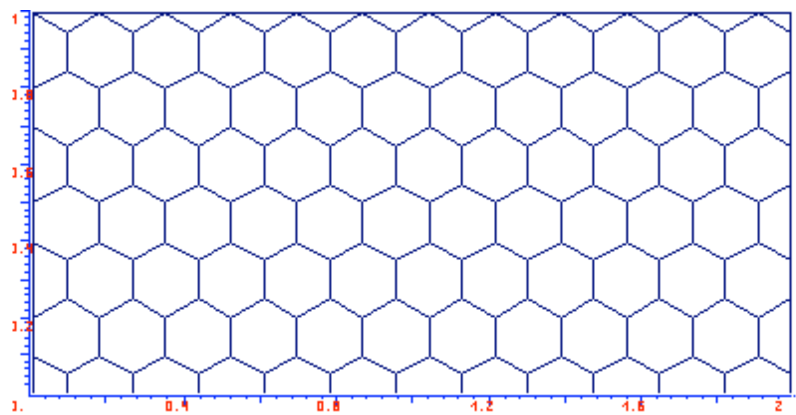
Fragment of the irregular grid, where are used algorithms regulating polygons

Examples of Computations

Problem 1.

Consider a cylinder with parameters $0 \le z \le 2$, $0 \le r \le 1$, $\alpha = 1.34$, $\beta = 2.25$, $\rho = 1$, $Q = 0$. It is required to determine the eigenvalues of parameter λ, which is a constant of particle variation in time.





Examples of Computations

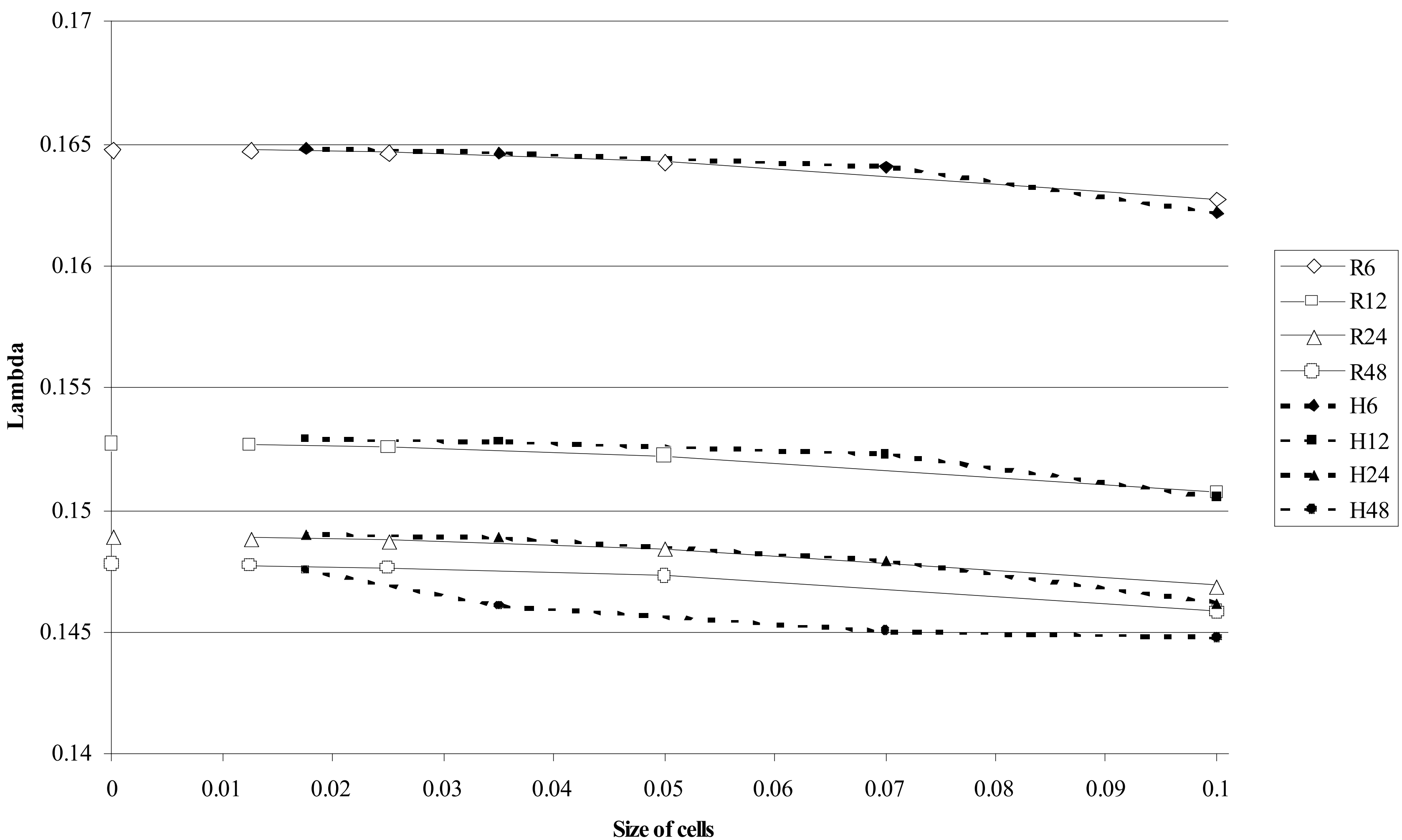
Table 1. The values of parameter

Rectangular	6	0.162753	0.164256	0.164631	0.164724
	12	0.150734	0.15222	0.15259	0.152683
	24	0.146903	0.148393	0.148762	0.148854
	48	0.145817	0.147295	0.147664	0.147756

Table 2. The values of parameter

Hexagonal	6	0.16219	0.16407	0.16464	0.164835
	12	0.150546	0.152302	0.152821	0.15292
	24	0.146242	0.147957	0.148874	0.149012
	48	0.14480	0.145054	0.146106	0.147558

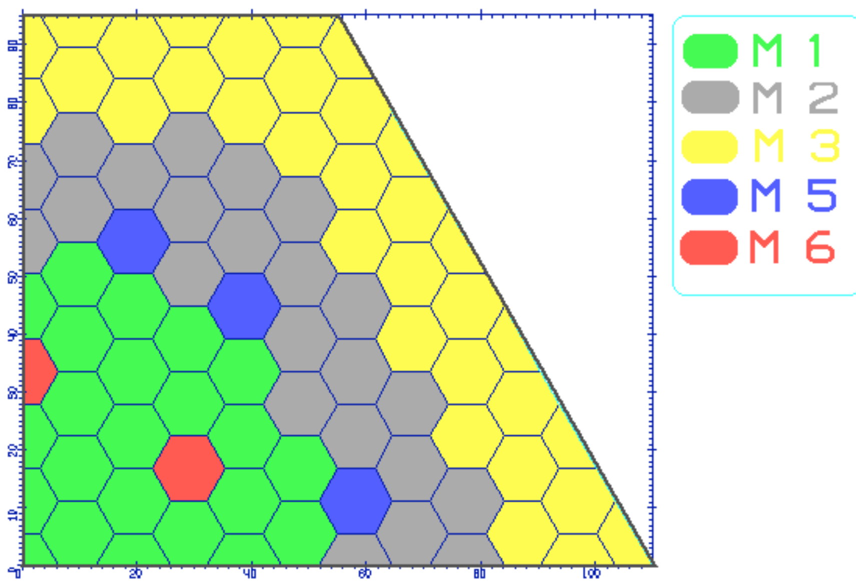
Examples of Computations



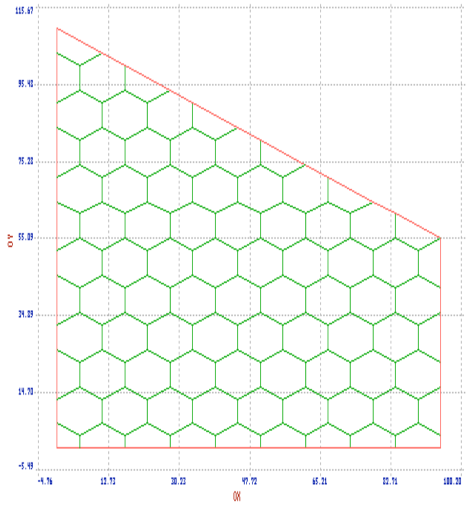
Examples of Computations

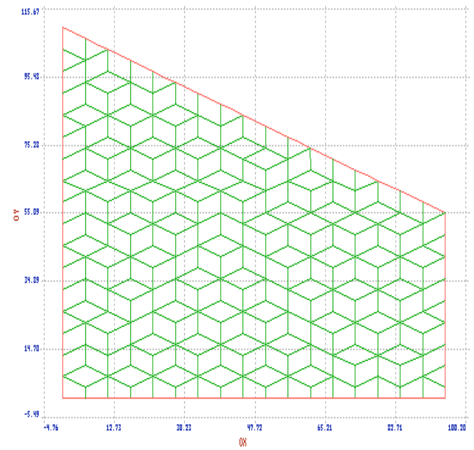
Problem 2.

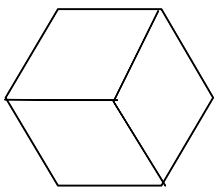
The task is to calculate the values of parameter k_{eff} in four-group approximation (Buckel et al.,1977).
Consider a horizontal section of the upper 3D half-reactor SNR-300.
Materials: 1,2 – fissionable materials, 3 - reflector, 5,6 –control rods.

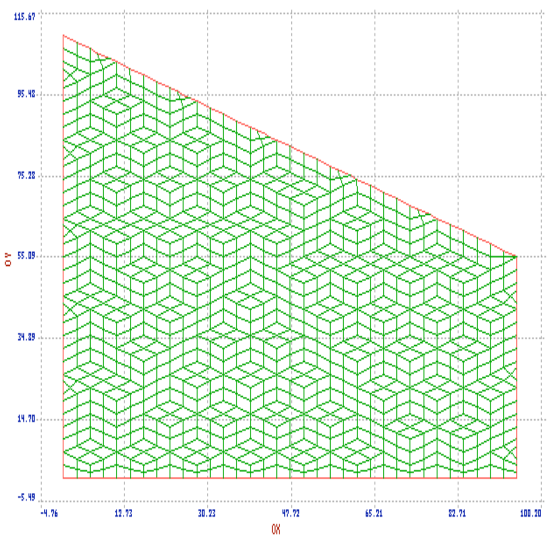


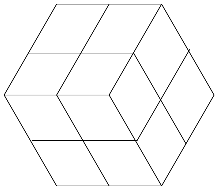
Examples of Computations

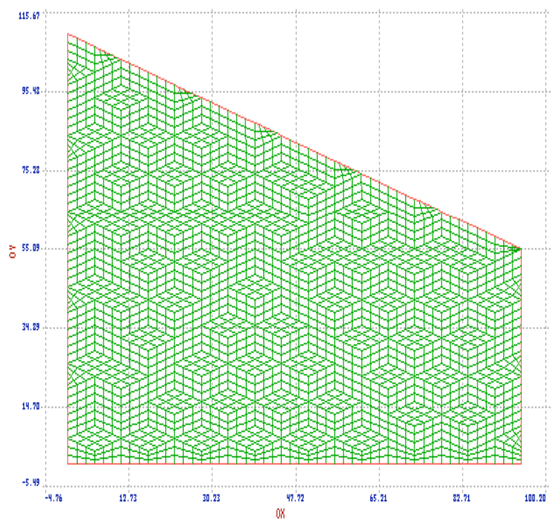


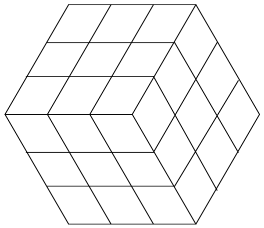












Examples of Computations

Table 4. The values of parameter k_{eff}.

Grid	k__
Hexagonal	1.13035
$\mathbf{L_{3}}$	1.13171
$\mathbf{L_{12}}$	1.13337
$\mathbf{L_{27}}$	1.13362
$\mathbf{L_{48}}$	1.13368

Conclusion

The practice of computations with the techniques of the transport equation simulation using polygonal spatial grids demonstrated a high efficiency of these algorithms. This is especially true for problems of complex geometries having local small-scale sub-regions, where application of polygonal cells allows efficient grid construction.