Modeling Material Failure with SPH

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J. Michael Owen

AX Division

Lawrence Livermore National Laboratory

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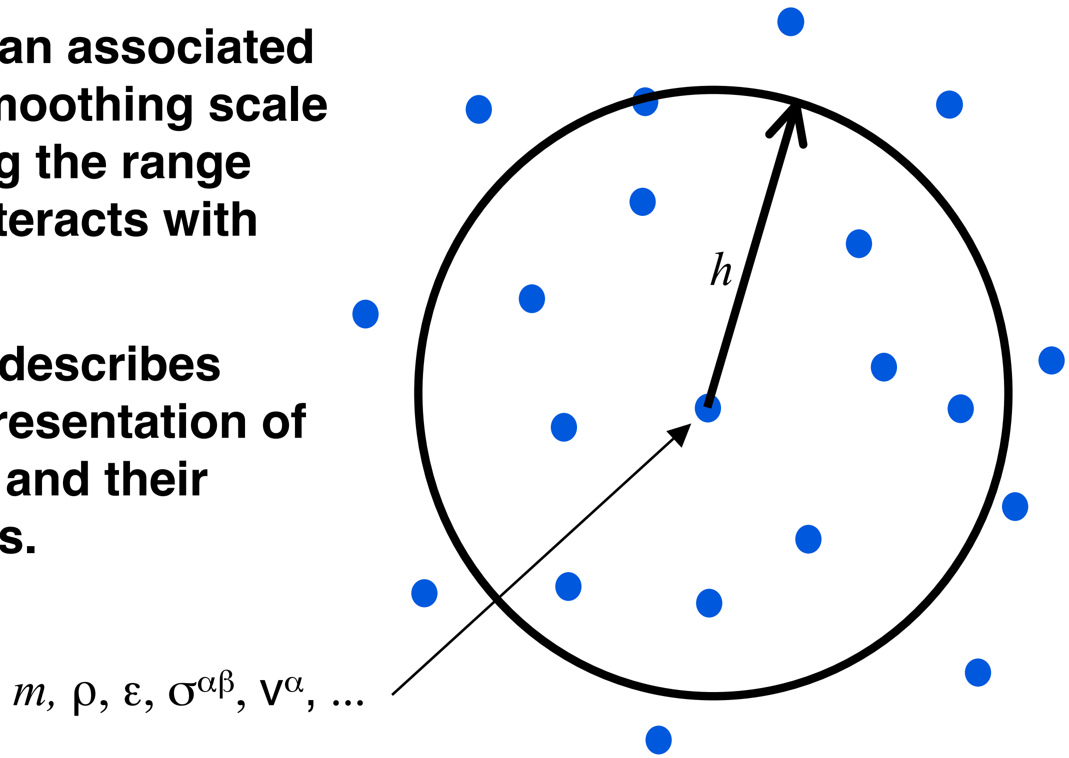
Lawrence Livermore National Laboratory, P.O. Box 808, Livermore, CA 94551–0808

Why is this interesting?

- Smoothed Particle Hydrodynamics (SPH) is the oldest of what is now a class of meshless hydrodynamic techniques.
- Pros:
  - Lagrangian *and* robust (no mesh to tangle).
    - Well suited for problems with complex flows or high deformation rates.
  - Simple to incorporate new physics.
    - History variables are not a problem – never any need to handle advection or remapping.
  - Naturally allows for gaps in material to form and open.
- Cons:
  - Sharp interfaces difficult to represent accurately.
  - More computationally expensive then mesh-based techniques.

Cartoon view of how SPH works.

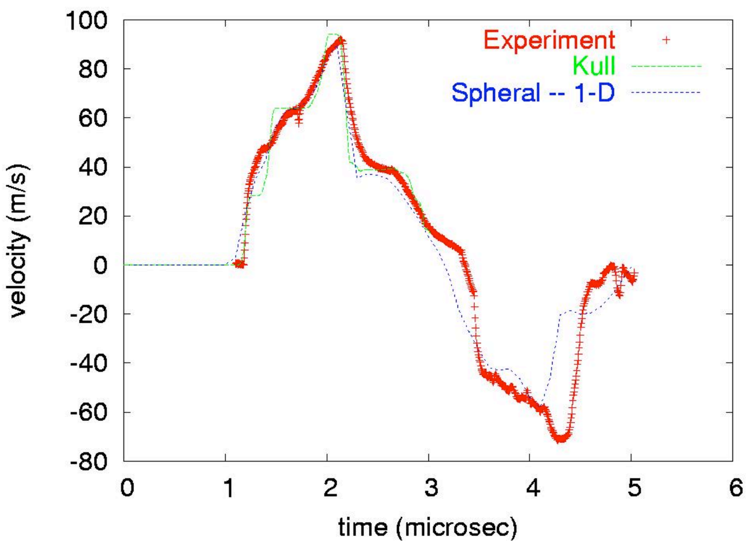
- **Physics variables defined at an arbitrary set of points in space.**
- **Points move with material velocity, arbitrarily reconnecting with new neighbors as simulation proceeds.**
- **Each point has an associated resolution or smoothing scale (*h*), representing the range over which it interacts with other points.**
- **SPH formalism describes continuous representation of nodal variables and their spatial gradients.**



Material Modeling.

- **SPH is traditionally applied to problems of compressible gas dynamics.**
- **However, it is simple to add solid material models.**
  - **I've added the Gruneisen EOS and Steinberg-Guinan rate independent strength model to my SPH code.**

Flyer plate impact experiment



Taylor anvil @ 150μsec

A Simple Material Failure Model.

- **Benz & Asphaug published a series of articles (1994, 1995, 1999) detailing a simple scalar damage model in an SPH code.**
  - **Statistical model of fracture based on the continuum model of Grady & Kipp (1980).**
- **We explicitly seed a set of flaw activation energies for each SPH node according to the Weibull distribution.**
  - **Number density of flaws having failure strains lower than ε is assumed to obey a power-law:**

$$n(\varepsilon) = k\varepsilon^m$$

- **Define the strain at node *i* (ε_i) based on the maximum eigenvalue of the tensile stress σ^t_i and Young's elastic modulus *E:***

$$\varepsilon_i = \frac{\sigma_i^t}{E}$$

Material Failure Model cont.

- If the strain at a node exceeds one of it's assigned flaw activation energies, then it accrues damage (*D* 2 [0,1]) at a rate

$$\frac{dD^{1/3}}{dt} = \frac{c_g}{R_s} = \frac{0.4c_l}{R_s}$$

  - c_g is the crack propagation speed
  - c_l is the longitudinal elastic wave speed.
  - R_s is the radius of the volume relieved by the crack, taken as a function of the resolution scale of the node *h*.

- The scalar damage *D* is used to create a new node of damaged material, dividing the mass between the original and damaged material as

$$m_i' = (1 - D_i)m_i,\ m_{D_i'} = D_i m_i$$

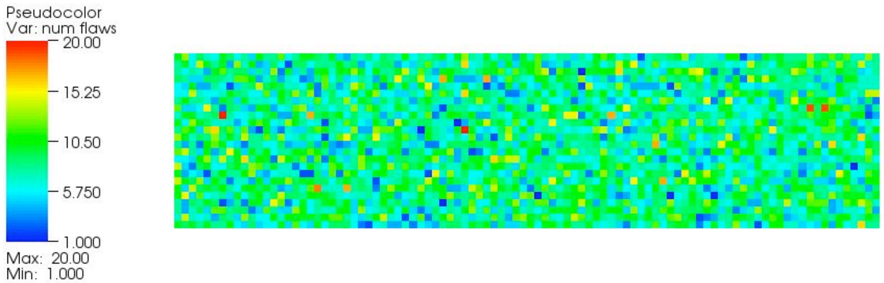
- The damaged material does not have strength and does not support tension.

Example: initial flaws in a steel rod.

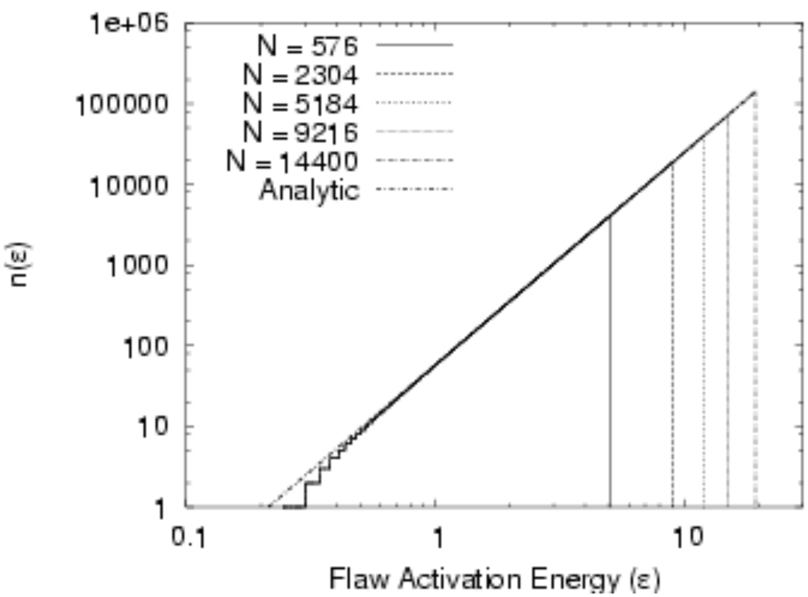
- **At problem setup we assign a population of flaw activation energies to each node:**

$$\varepsilon_{i,j}^{\mathrm{act}} = \left[\frac{j}{kV}\right]^{1/m}, \; j \in [1, N_f]$$

Number of flaws assigned to each node



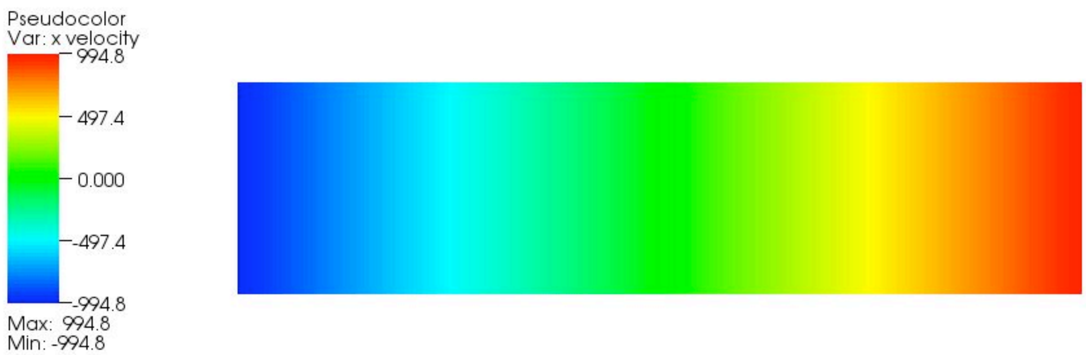
Distribution function of flaw activation energies



Example: Tensile steel rod.

- Take a 20x5 cm steel rod with initial velocity $\mathbf{v}(x) = \begin{pmatrix} v_0 x \\ 0 \end{pmatrix}$
  - Creates initially constant strain rate throughout the rod.
- Enforce constant velocity on the ends of the rod.
  - Forces the ends of the rod to draw outward, regardless of the tensile strength of the material.

x velocity, t = 0 μsec



Expected fragment size.

- **Grady & Kipp compute an expected fragment distribution based on the population of flaws which activate and grow to a damage *D* = 1.**
  - **For a constant strain rate** $\dot{\varepsilon}$

$$L(\dot{\varepsilon}) = \frac{6c_g}{m+2}\alpha^{(m+3)^{-1}}\dot{\varepsilon}^{-m/(m+3)}$$

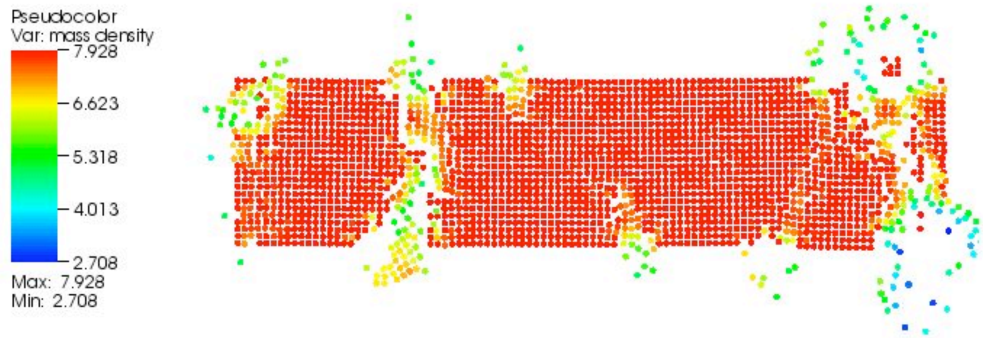
$$\alpha = \frac{8\pi c_g^3 k}{(m+1)(m+2)(m+3)}$$

- **In the following tensile rod examples, for v_0 = 10 m/sec and v_0 = 100 m/sec we should expect typical fragment sizes of 8.6 cm and 2.9 cm, respectively.**
  - **This implies a 20 cm rod should break completely across in 2–3 places for v_0 = 10 m/sec, vs. 6–7 places for v_0 = 100 m/sec.**

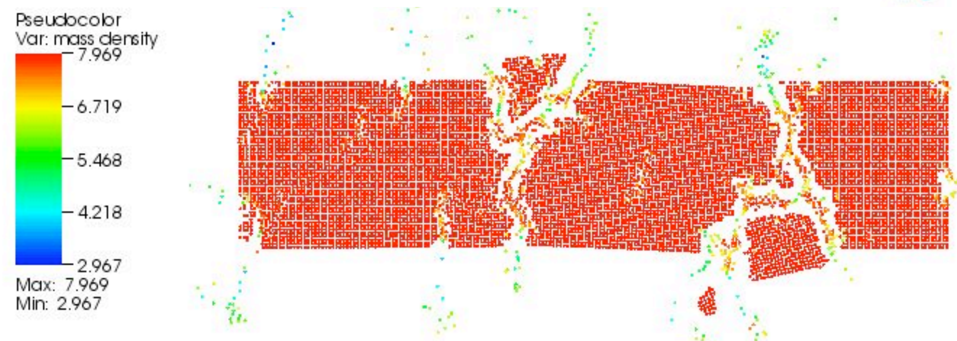
Tensile steel rod: v_0=10 m/sec @ t=500 μsec

mass density

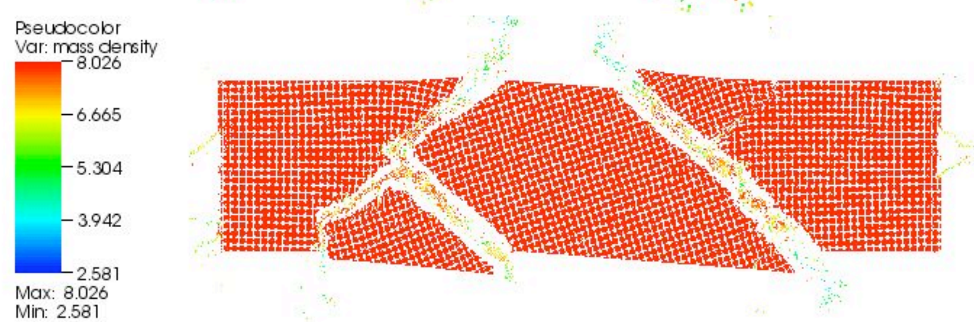
Resolution: 100x25



Resolution: 200x50



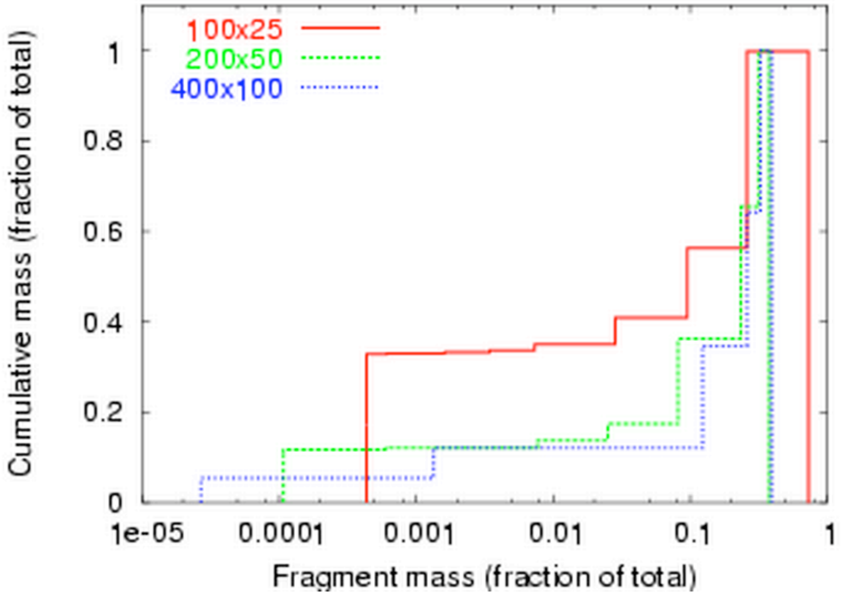
Resolution: 400x100

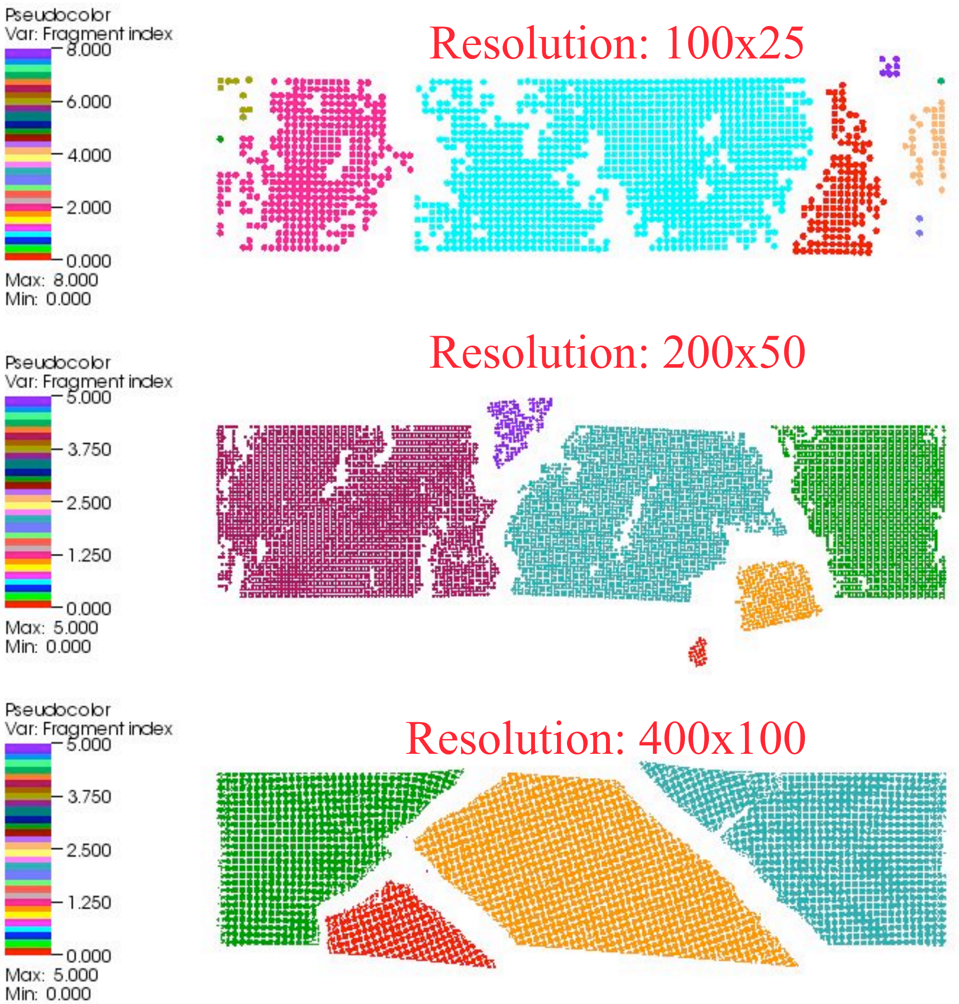


Tensile steel rod: v_0=10 m/sec @ t=500 μsec

Fragment properties

Fragment mass distribution function

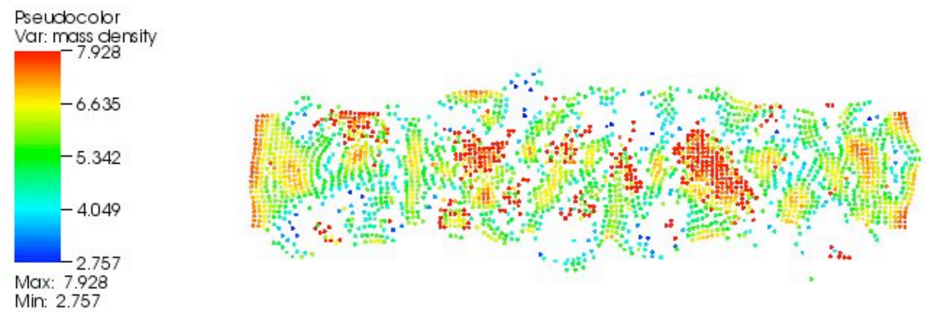




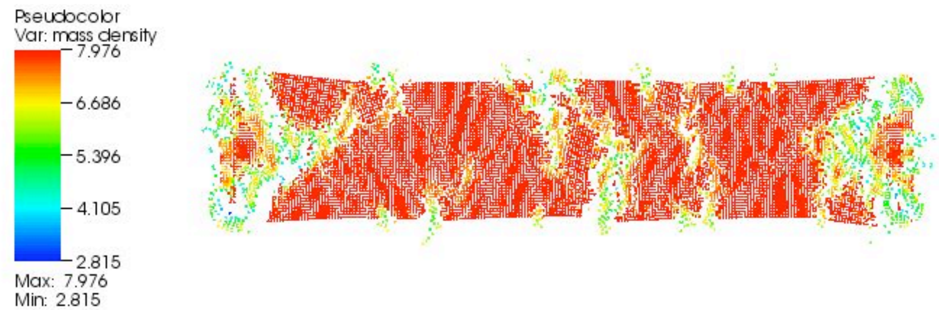
Tensile steel rod: v_0=100 m/sec @ t=500 μsec

mass density

Resolution: 100x25



Resolution: 200x50

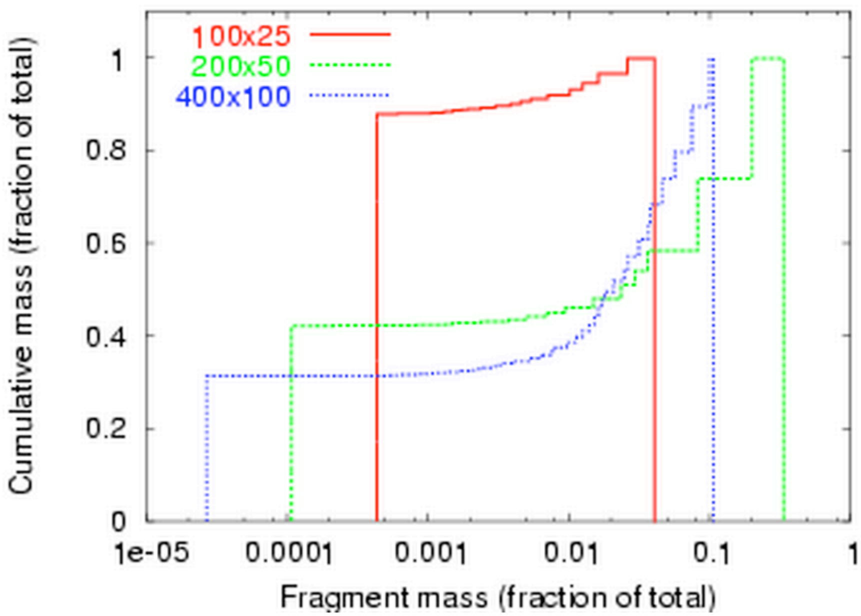


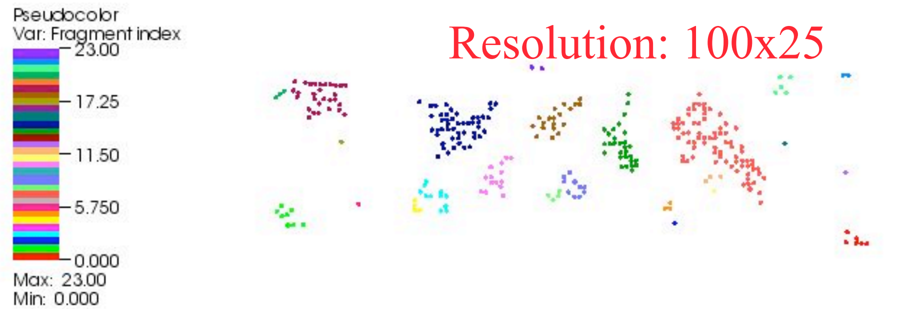
Resolution: 400x100

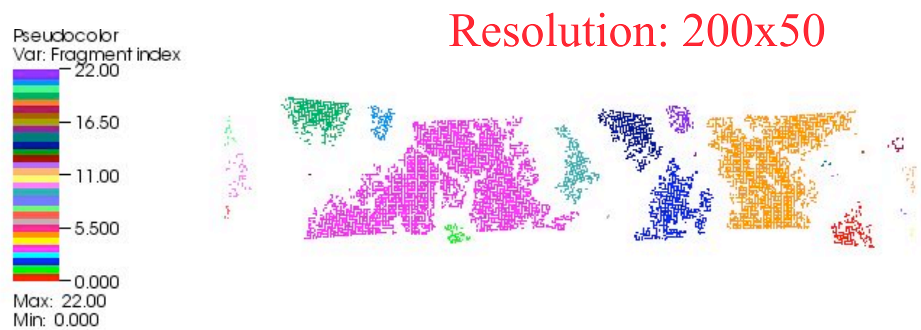
Tensile steel rod: v_0=100 m/sec @ t=500 μsec

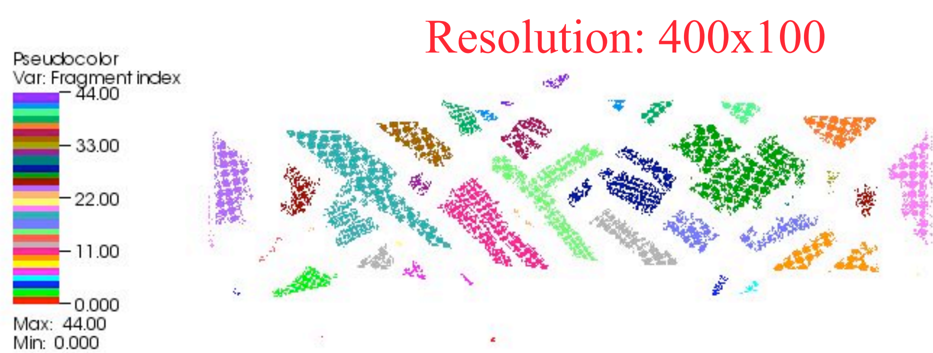
Fragment properties

Fragment mass distribution function









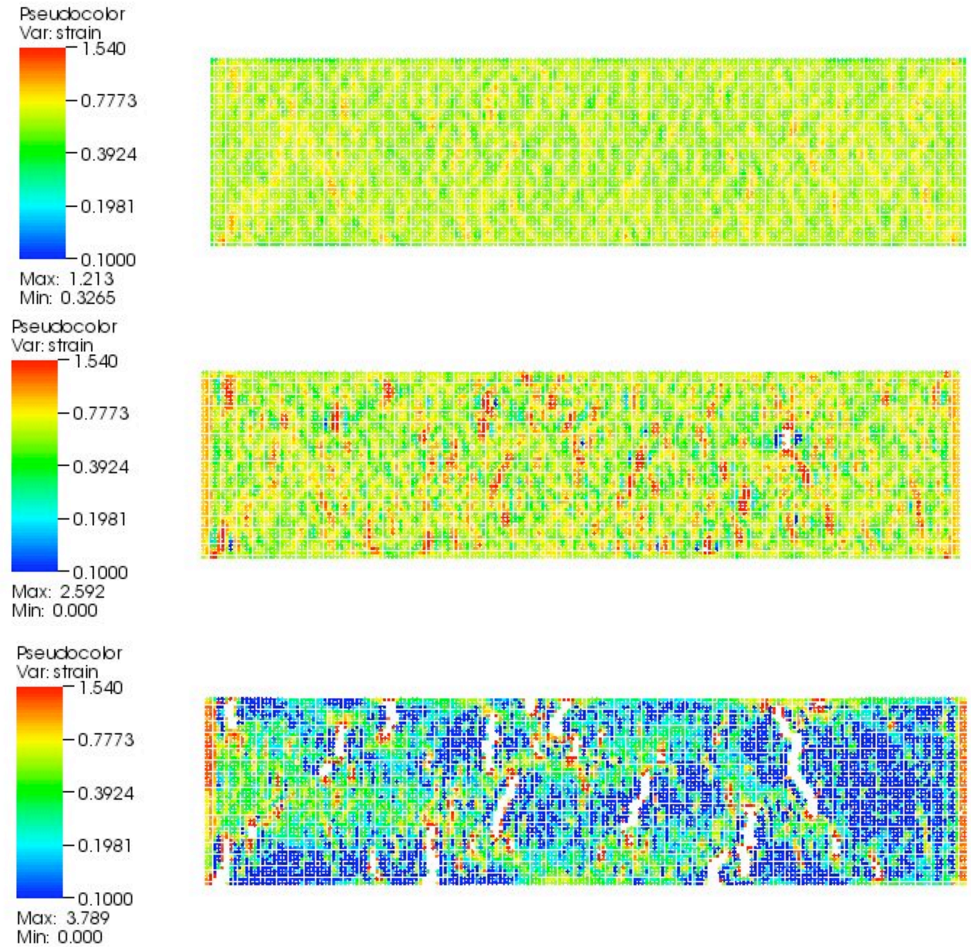
Tensile steel rod: v0=10 m/sec — strain

- **So why do these rods break where they do?**

t = 37.5 μsec

t = 62.5 μsec

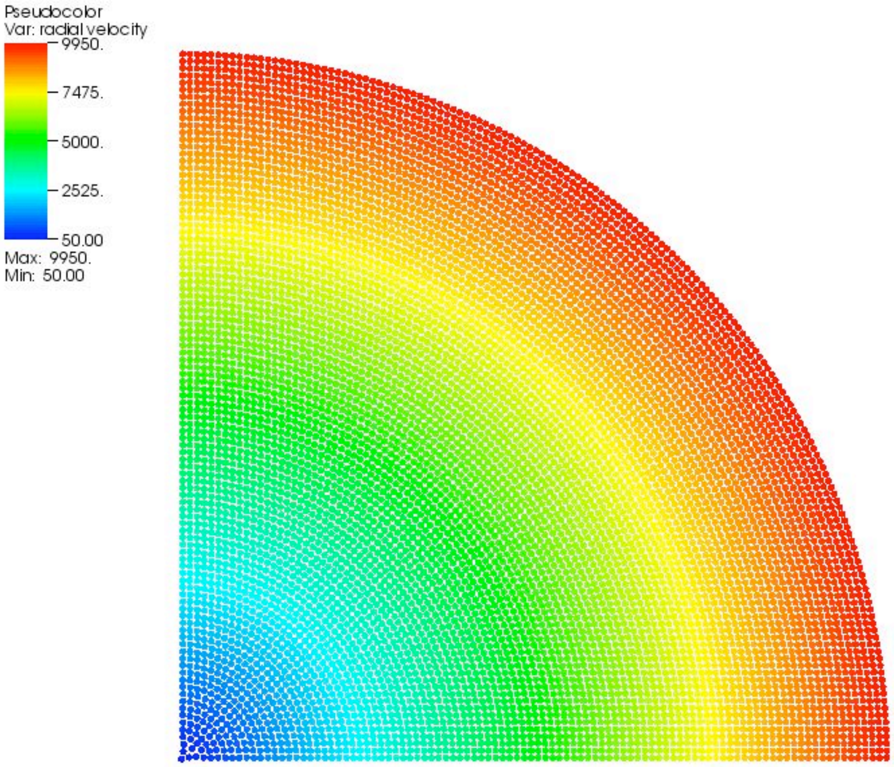
t = 100 μsec



Tensile steel disk

- **Now consider imposing an initial velocity field on a 2-D steel disk of** $v_r = v_0 r$

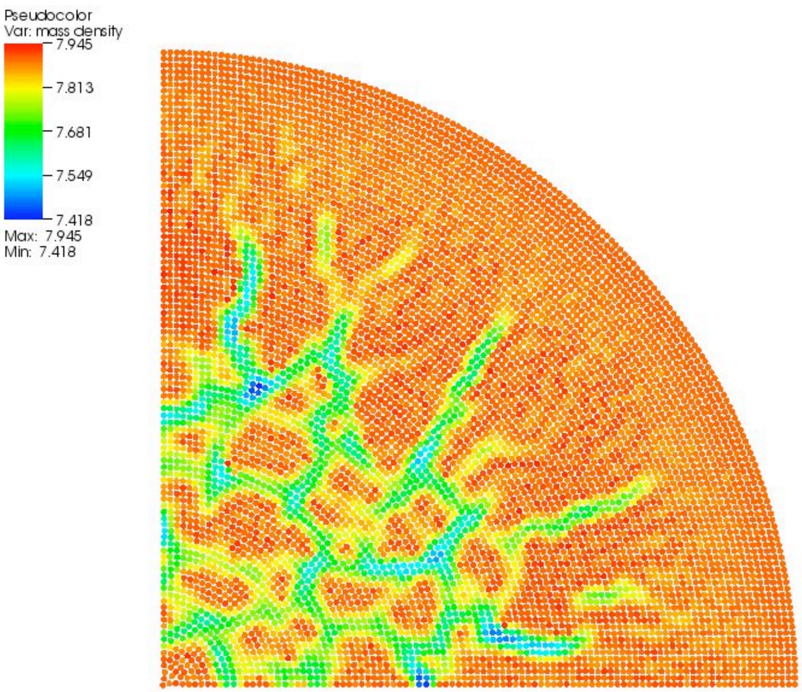
- **In this example:**
  - **v_0 = 100 m/sec**
  - **Simulate one quadrant**
  - **No forced velocity on outer boundary**



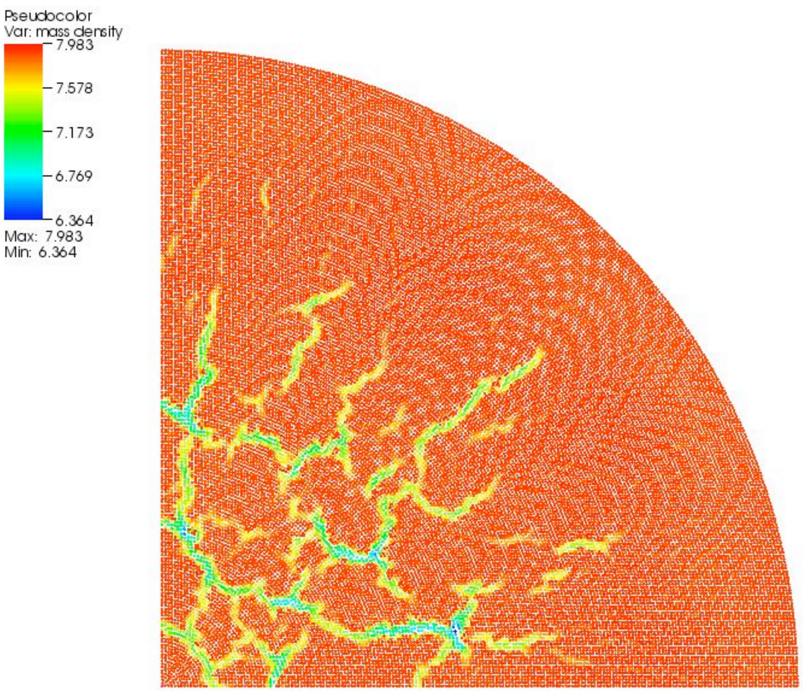
Tensile steel disk @ t = 25 μsec: mass density

- These plots show all materials (damaged and undamaged).

$n_r = 100$



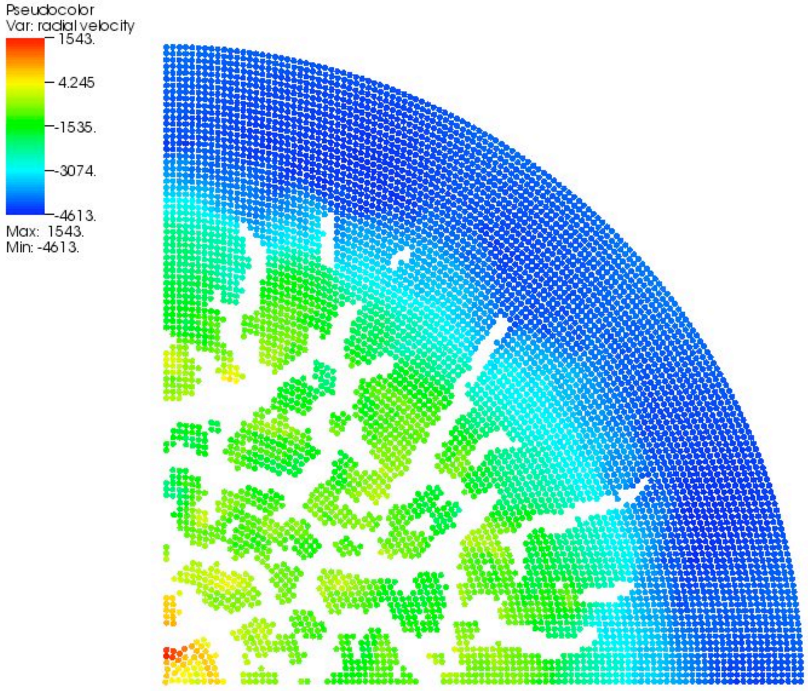
$n_r = 200$



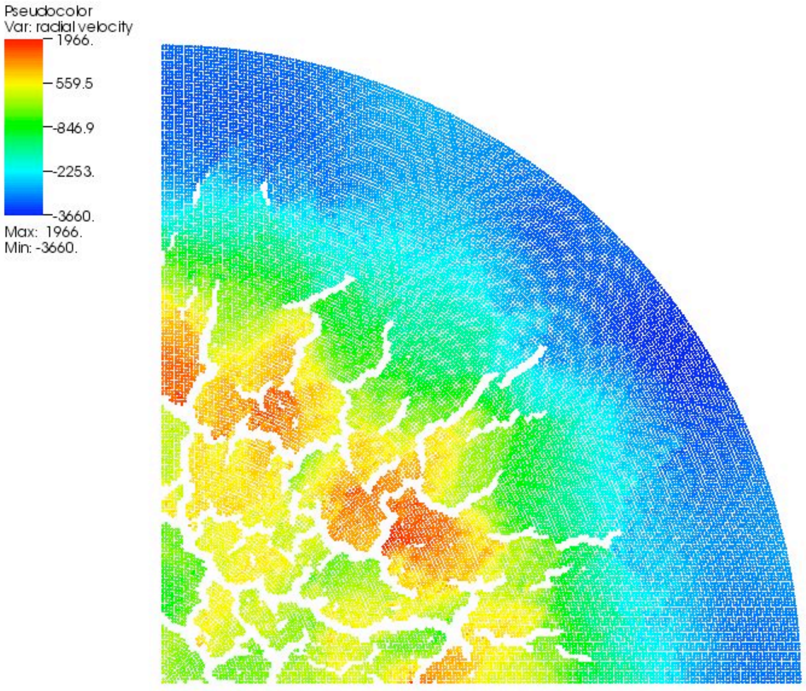
Tensile steel disk @ t = 25 μsec: radial velocity

- **Plotting undamaged material only, at two different resolutions.**
- **Note outermost radii have turned around by this time.**

$n_r = 100$

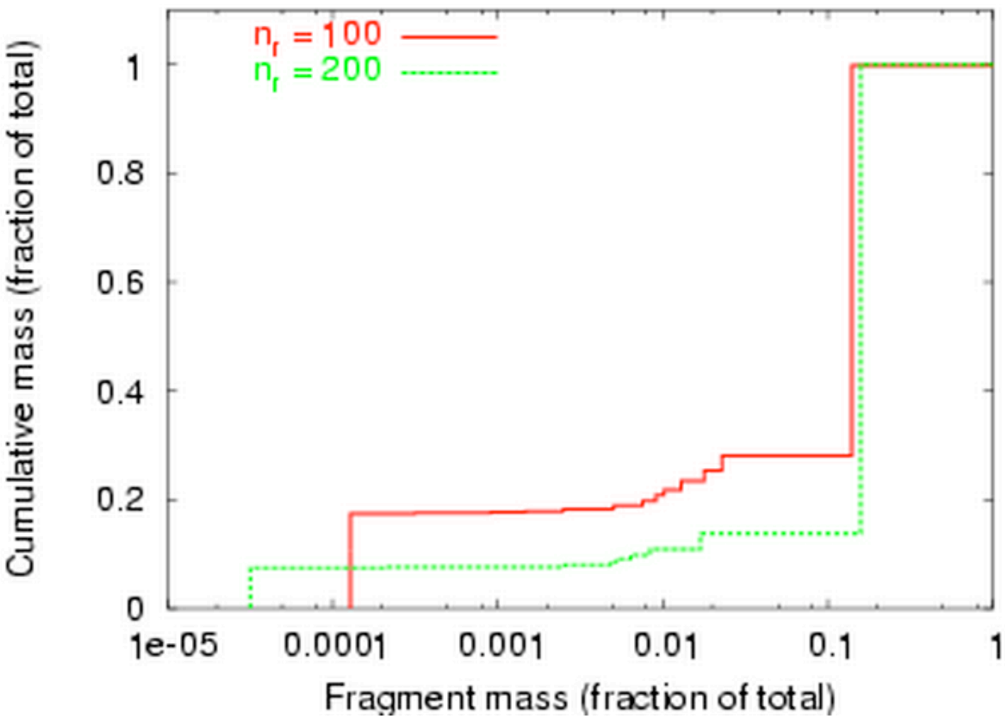


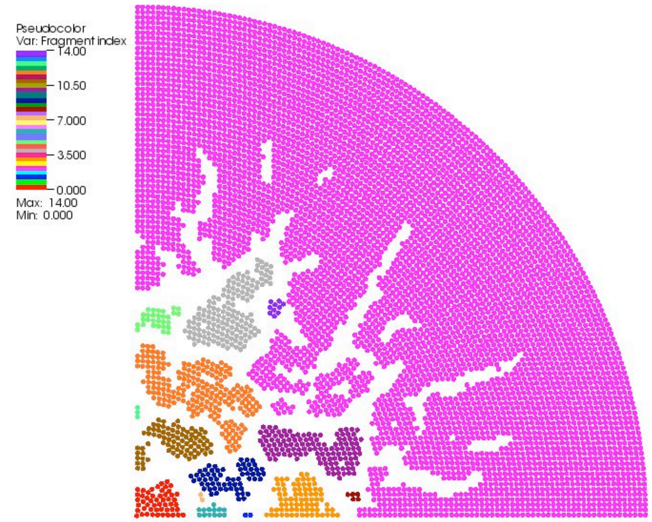
$n_r = 200$

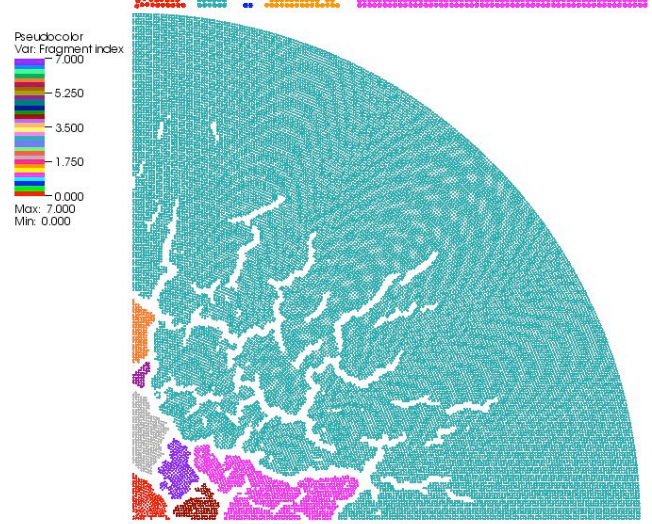


Tensile steel disk @ t = 25 μsec: fragments

Fragment mass distribution function



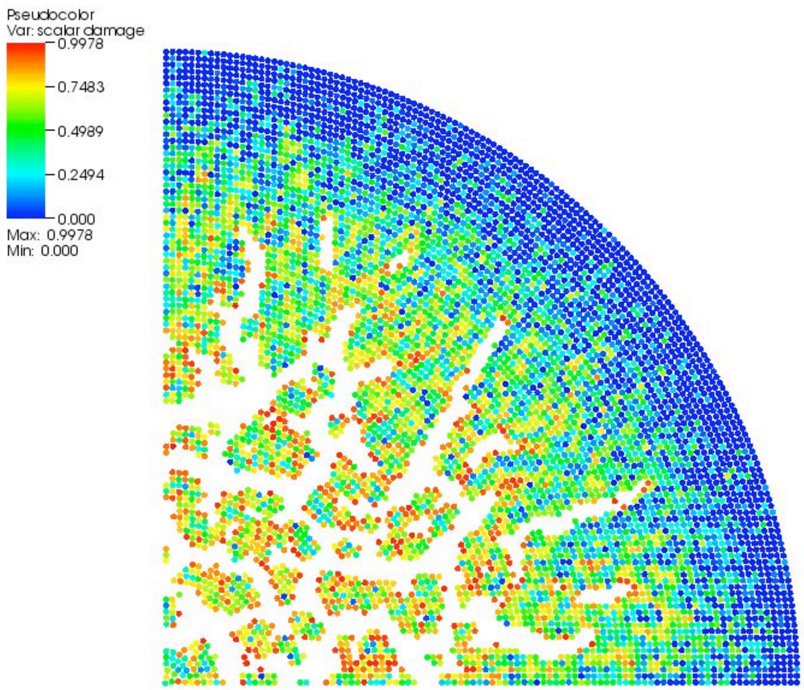




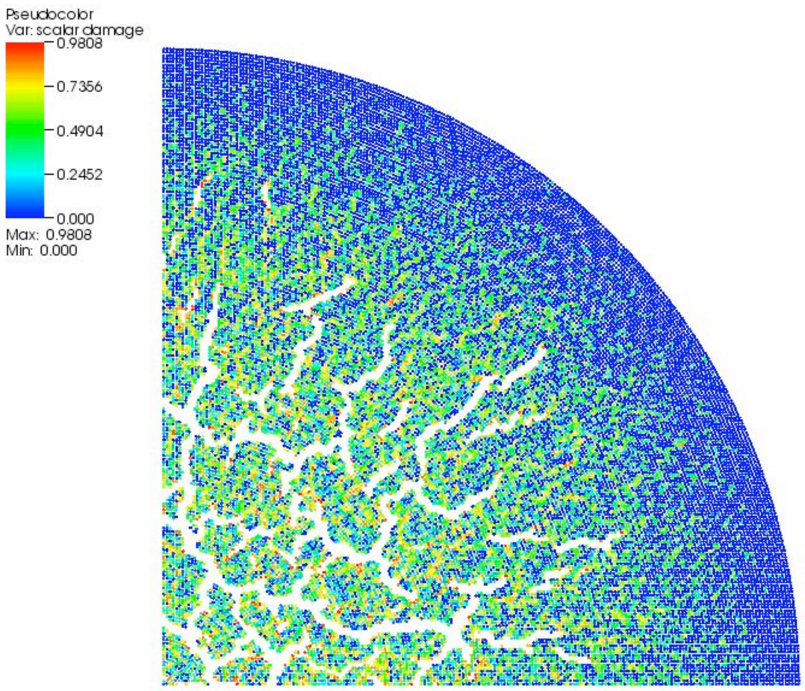
Tensile steel disk @ t = 25 μsec: damage

- **Plotting undamaged material only.**
- **Note partially damaged material spread throughout disk.**

$n_r = 100$

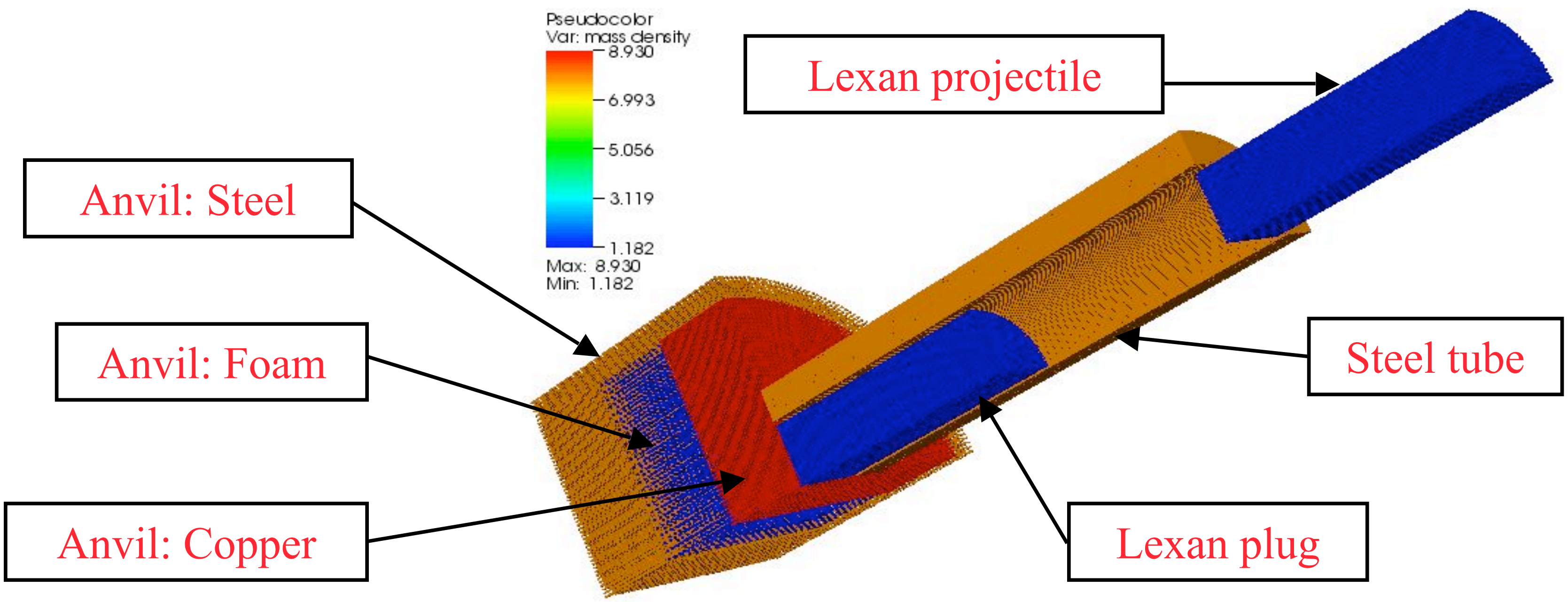


$n_r = 200$



Expanding tube gas gun experiment

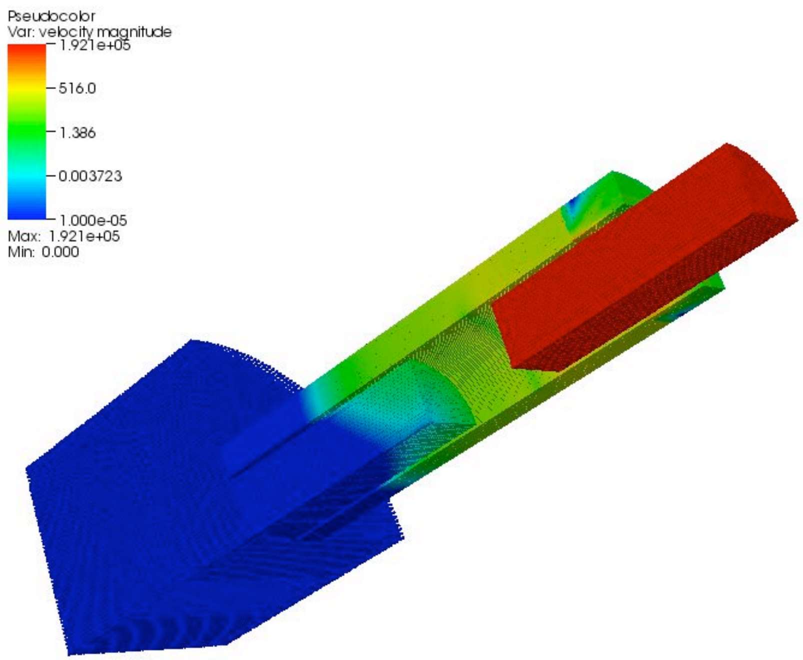
- **Model a series of experiments fragmenting metal tubes due to the impact of a plastic projectile within the tube.**
  - **Vogler TJ, Thornhill TF, Reinhart WD, Chhabildas LC, Grady DE, Wilson LT, Hurricane OA, & Sunwoo A, "Fragmentation of Materials in Expanding Tube Experiments,"** ***Int. J. Impact Engng*****, 2003; 29:735–746**



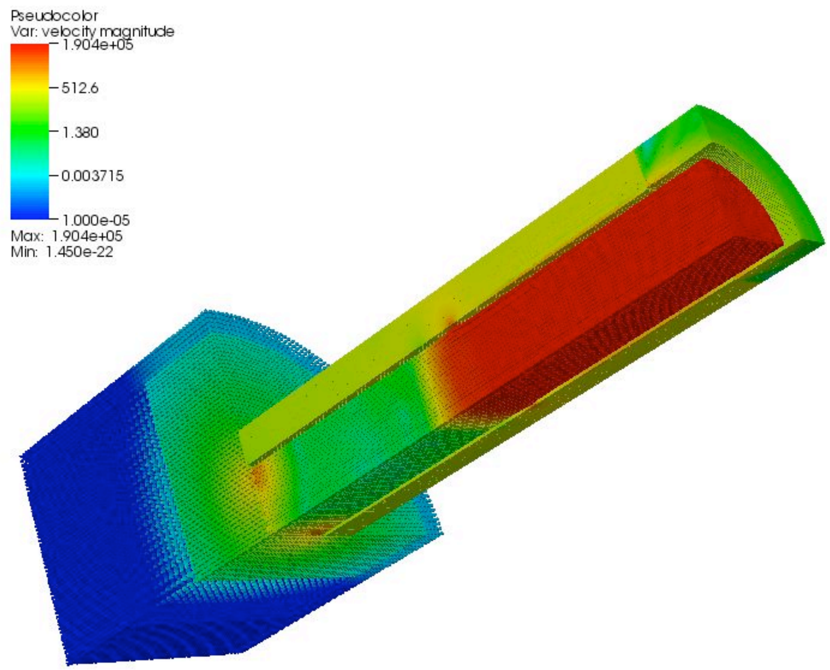
Expanding tube – early evolution

- **Velocities during projectile entry.**

8.0 μsec

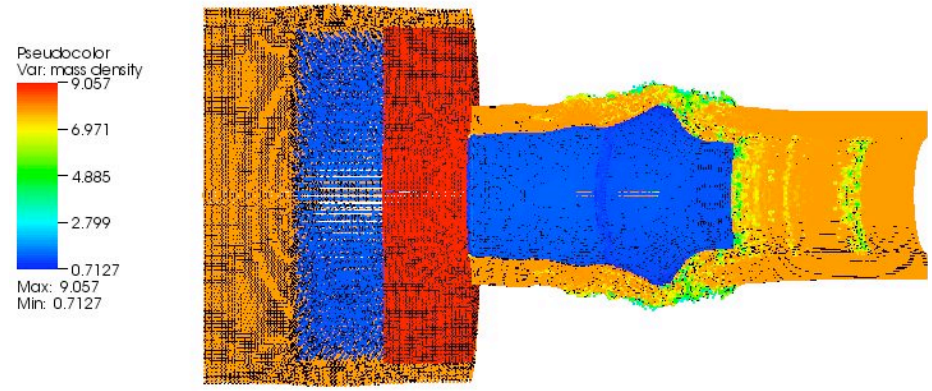


13.75 μsec

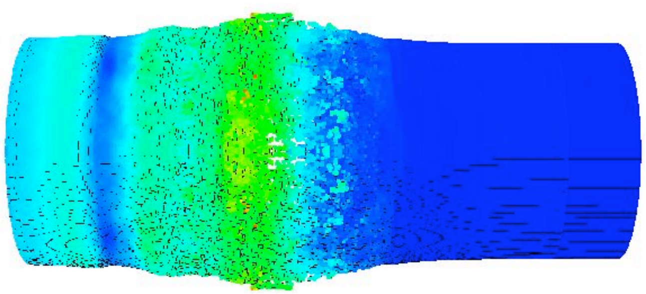


Expanding tube – tube during early expansion

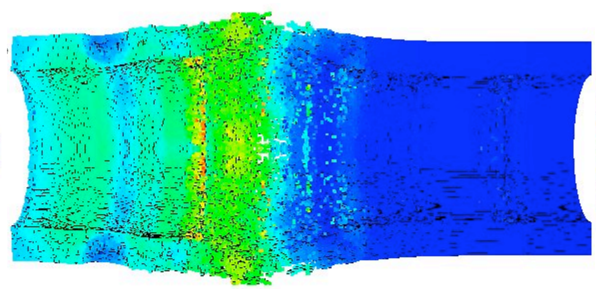
- **At 26.75 μsec, projectile & plug are pushing out against the tube, causing extensive damage at the expansion point.**



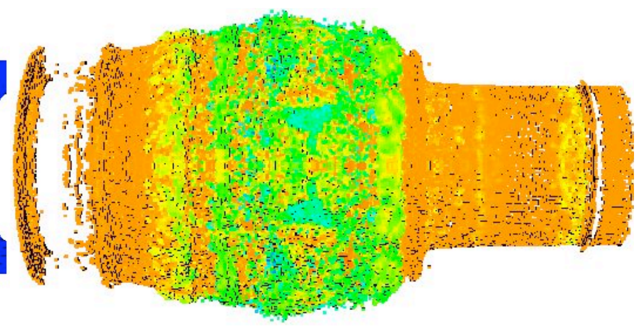
Exterior, undamaged material, velocity



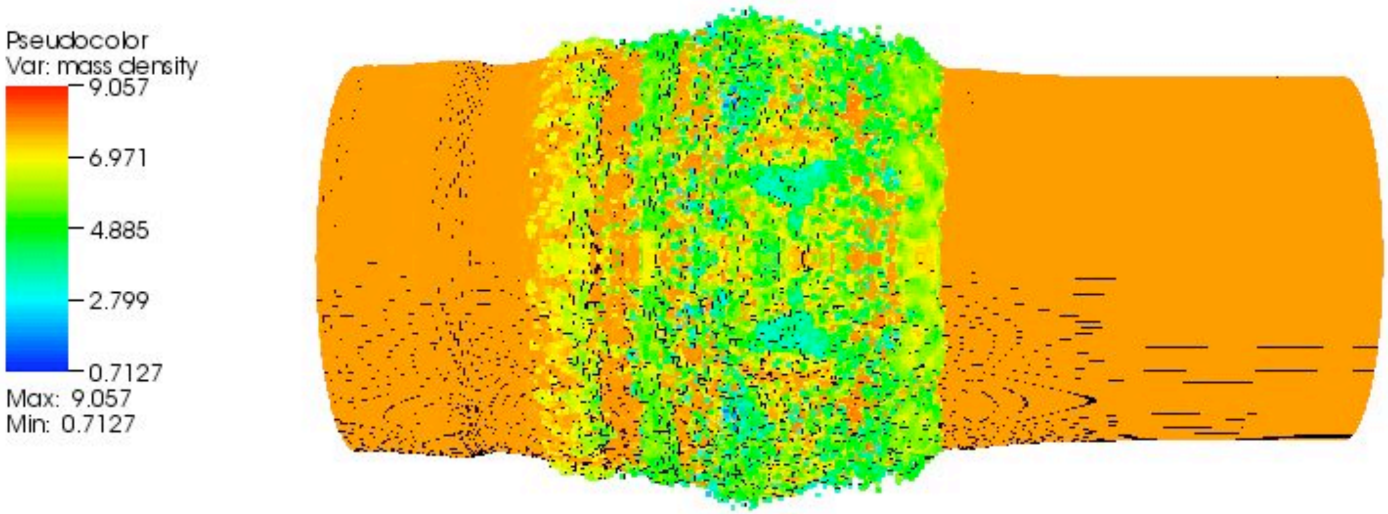
Interior, undamaged material, velocity

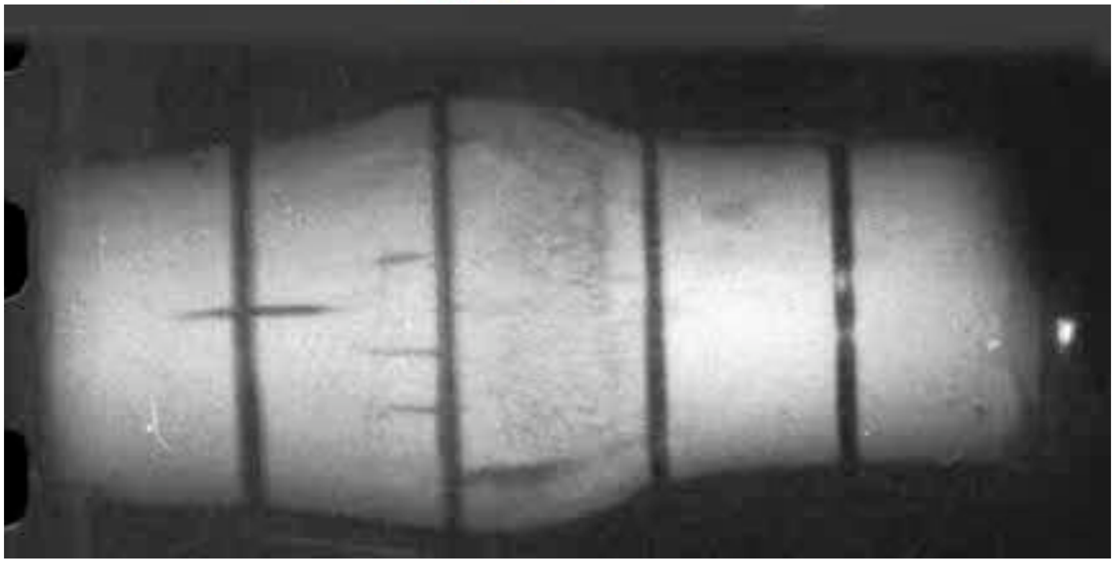


Exterior, damaged material, mass density



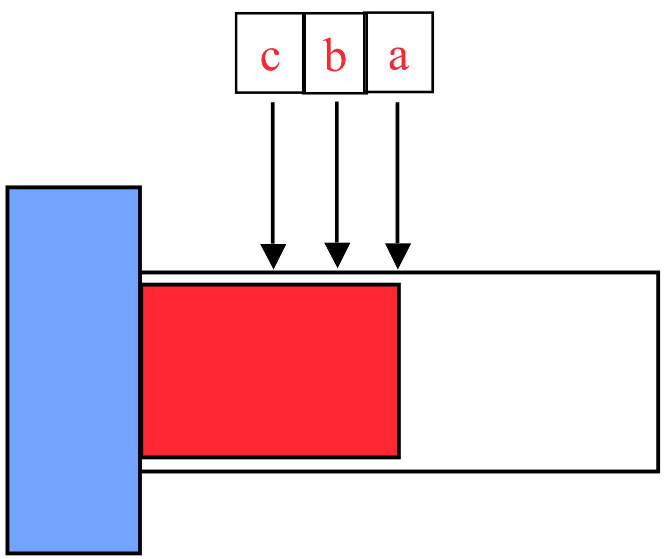
Expanding tube – experiment results.

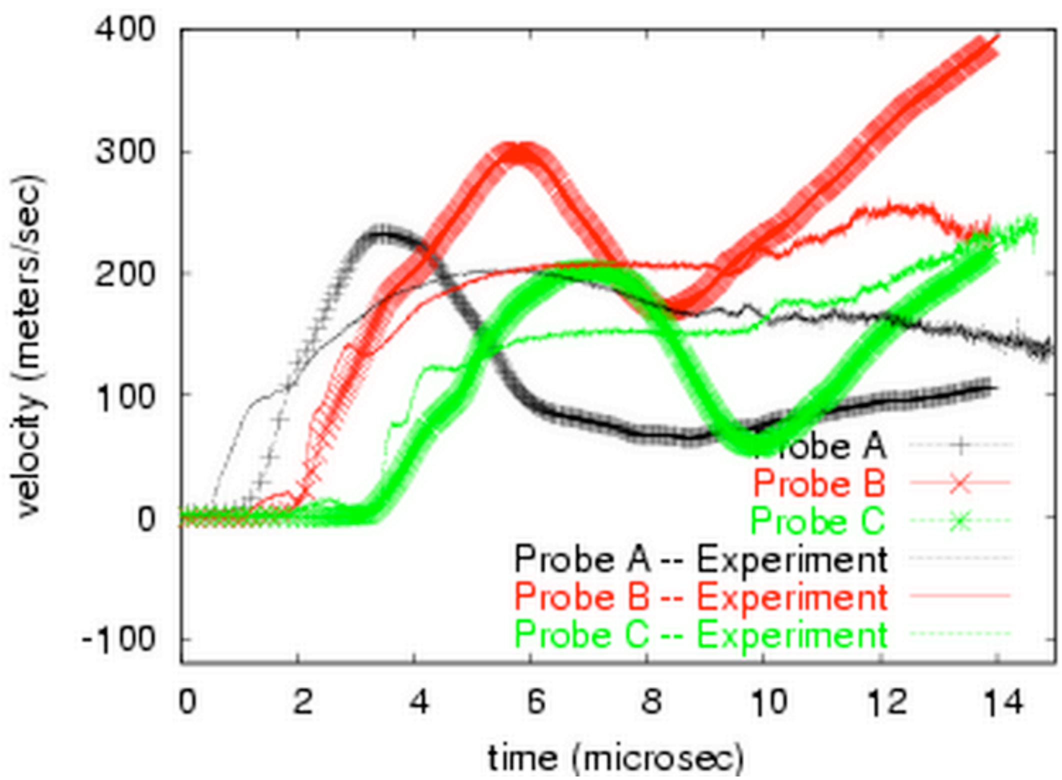




Expanding tube – VISAR data

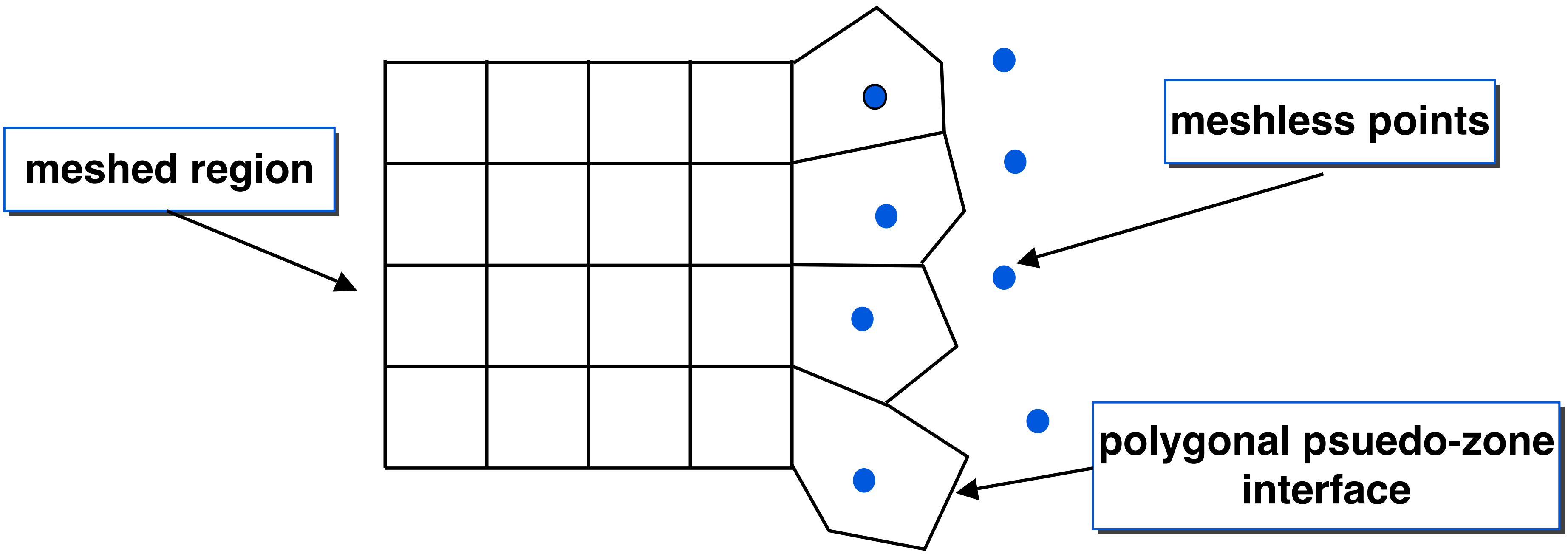
- This experiment was instrumented with three VISAR velocity probes on the exterior of the tube.
  - Probe A @ 25 mm from anvil
  - Probe B @ 20 mm
  - Probe C @ 15 mm





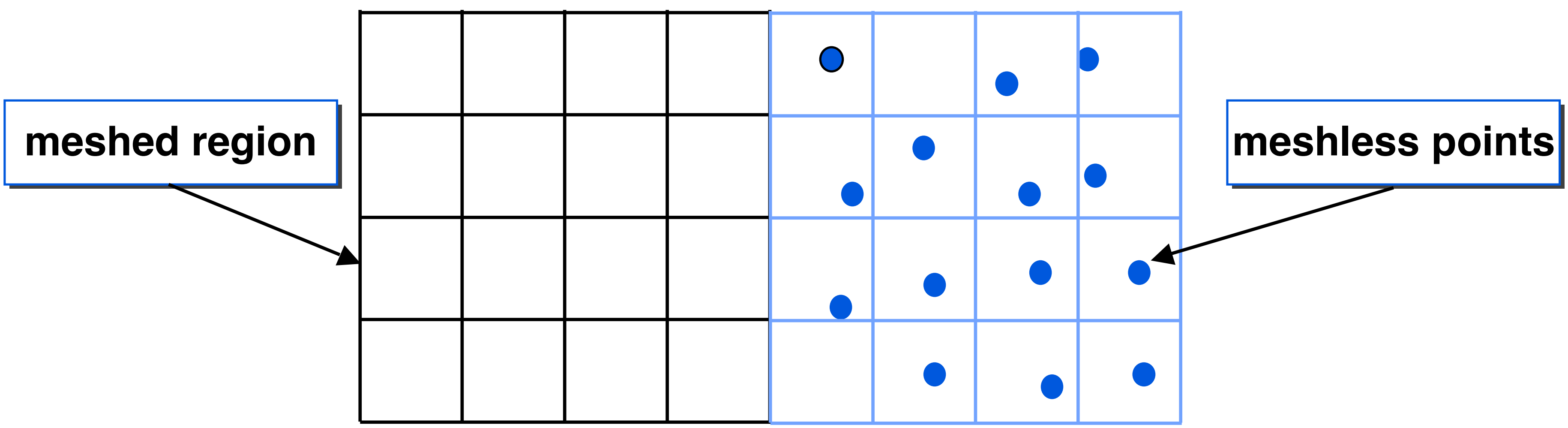
How might we couple to a mesh-based code?

- There are a few possible routes to coupling a meshless scheme such as SPH with a traditional mesh-based code:
  - Direct hybridization, possible with a code that can use unstructured polygonal elements (or potentially triangular)



How might we couple to a mesh-based code?

– Interpolating data back and forth for overlapping mesh and meshless regions.
  - Properties for meshless region solved on meshless points, and then mapped back onto the mesh



Things missing/remaining to be done.

- Apply current techniques to a wider variety of interesting experiments.
- Benz & Asphaug failure model only accounts for tensile failure, need to follow shear failure as well.
  - Can enhance current algorithm, implement other more advanced models (Johnson-Cook, MARFRAC, etc.)
- Fragments should be identified during the course of a run and spun off as new materials.
  - Prevent strength from operating between fragments that happen to run into one another.
- We really should follow where melting occurs, and reset both failed and undamaged material if/when refreezing.
- Assorted numerical improvements:
  - Improved surface treatment of variables and gradients, summed mass density, ASPH, …