Lagrangian Modeling at Sandia National Laboratories: Current Status and Future Directions

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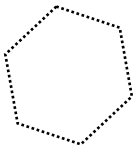
Lagrangian Simulation

- Lagrangian modeling including contact modeling is highly effective for modeling complex solid structures undergoing significant but not extreme elastic and plastic deformations.
- Key Challenge
  - Historical emphasis on mean gradient quadrilateral and hexahedral discretizations.
  - Hexahedral mesh generation is very expensive in man time.
  - Alternative discretization strategies are needed for tetrahedral meshes.
- We will give an overview of two approaches attacking these issues.

Node-Based Uniform Strain Elements (Nodal Based Tet (NBT))

- **Presentation follows Dohrmann, et al., Int J. Num Methods in Eng., 47:1540-1568 (2000)**
- **Triangles, J; Nodes, J_k, I, L**

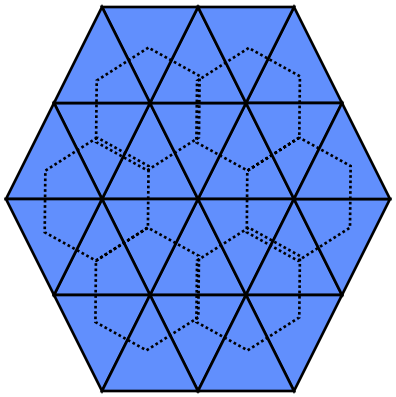
$$B_{Jik} = \partial A_J / \partial x_{iJ_k} \qquad A_J = x_{iJ_k} B_{Jik}$$



$$1 = \alpha_{J_1 J} + \alpha_{J_2 J} + \alpha_{J_3 J}$$

$$\widehat{A}_L = \alpha_{LJ} A_J \qquad \widehat{B}_{LiI} = \partial \widehat{A}_L / \partial x_{iI}$$

$$f_{iI} = \sigma_{ij}^{L} \widehat{B}_{LjI}$$



Displacements and stresses are both centered at the nodes of original mesh (centers of hexagons) which gives an optimal constraint ratio.

Large stencil

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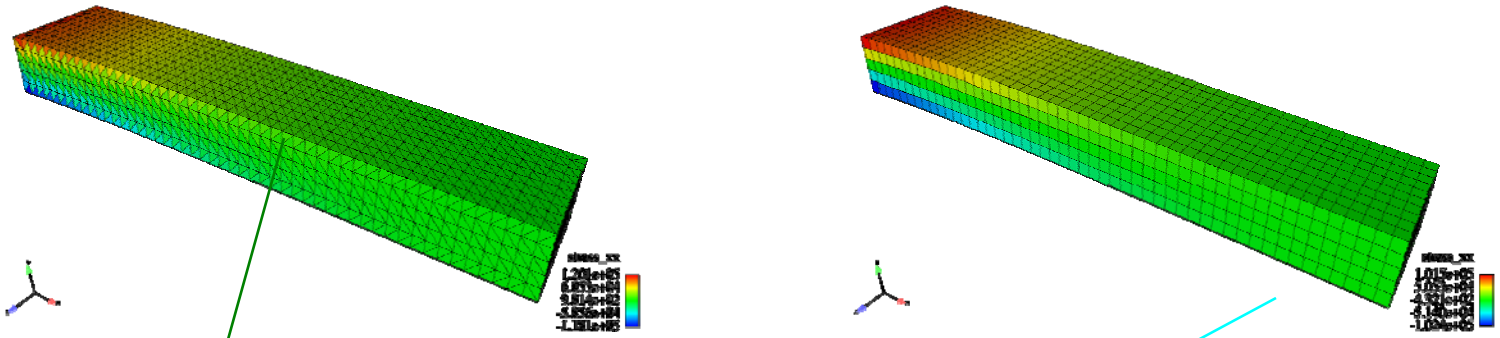
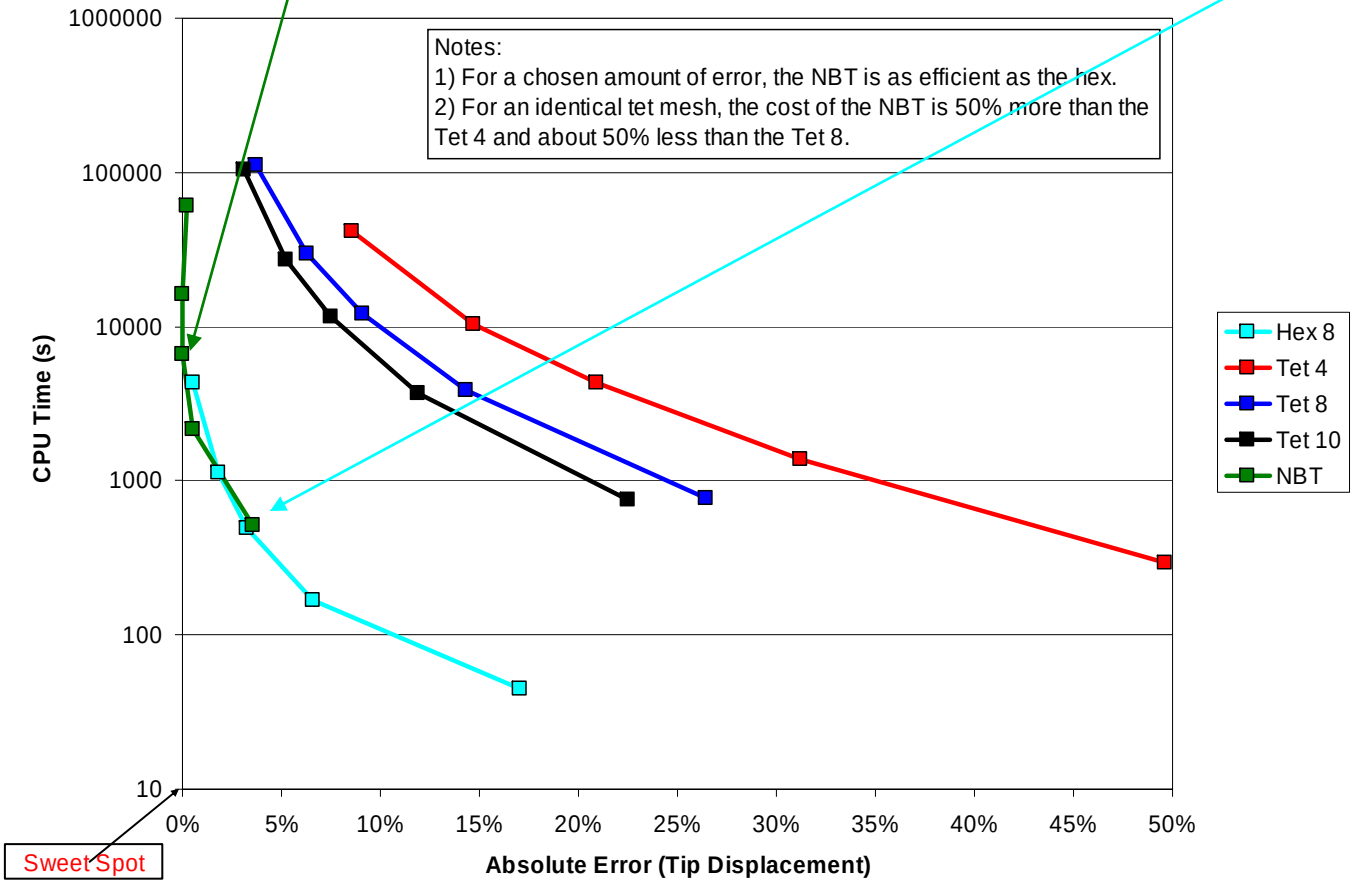
Performance Comparison on Bending Beam





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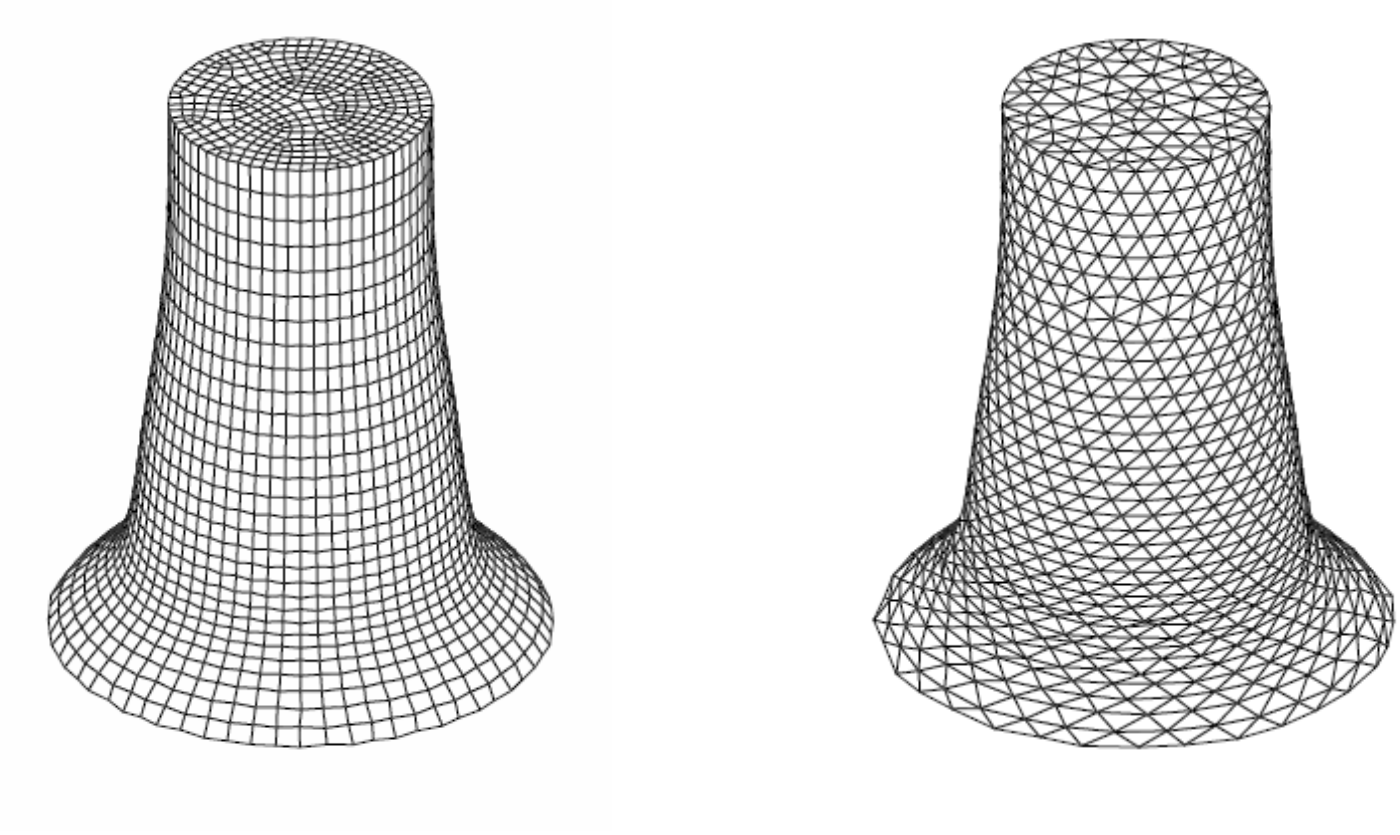
Taylor Impact – Hex8 versus Nodal Based Tet



Nodal based tet performs well on this basic litmus test.

Examples courtesy of S.W. Key, J.D. Gruda, and A.S. Gullerud

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Nodal based tets - Experience

- Elements work well without locking.
- Apparent super-convergence away from the boundary.
- More expensive than hexahedral elements but still competitive.
- These elements may permit rapid turnaround of computations when meshing the geometry is the major bottleneck.

SUPG Stabilized Shock Hydrodynamics

- Overview of ongoing Streamline-Upwind Petrov-Galerkin hydrodynamics research
  - Scovazzi (Sandia), Mark A. Christon (Los Alamos National Laboratory), Thomas J.R. Hughes (Institute for Computational Engineering and Sciences (ICES), University of Texas at Austin)

- Reliable simulations on simplex-type meshes (2D-triangles/3D-tetrahedra)
  - No artificial stiffness, no locking

- Based on SUPG concept (*Brooks and Hughes, CMAME, 1982*):
  - *Stable, upwinded, multiD* formulation for advection-diffusion systems
  - Variationally consistent, effectively a *Petrov-Galerkin* formulation
  - *Optimal error estimates* in both diffusion and advection limits
  - Applied to compressible Navier-Stokes in Eulerian coordinates (1986-1995, Hughes *et al.*)

- Lagrangian hydrodynamics formulation, 2nd order in space/time:
  - Quadrilateral and *triangular* elements in 2D
  - Globally conservative formulation, locally conservative in a weak sense
  - All variables node-centered $\Rightarrow$ no pressure gradient reconstruction issues
  - Continuous in-space-time trial space: piecewise linear-in-space-time
  - Discontinuous-in-time/continuous-in-space test space: constant-in-time/linear-in-space
  - Space-time discretization + single point quadrature in time $\Rightarrow$ mid-point rule in time
  - Predictor/multi-corrector approach using a fixed-point nonlinear iteration
  - Customized SUPG stabilization $\Rightarrow$ importance of Galilean invariance
  - Noh-type viscosity $\Rightarrow$ no overheating

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SUPG Stabilized Shock Hydrodynamics

- **1D pure advection:** $c\,\partial\phi/\partial x = f, \quad c < 0$

From central to upwinded differences using a numerical viscosity:

$$c\frac{\partial\phi}{\partial x} \cong c\left(\frac{\phi_{n+1}-\phi_n}{\Delta x}\right) = c\left(\frac{\phi_{n+1}-\phi_{n-1}}{2\Delta x}\right) + \boxed{\frac{c\,\Delta x}{2}\left(\frac{\phi_{n+1}-2\phi_n+\phi_{n-1}}{\Delta x^2}\right)}$$

- **SUPG consistency:** Perturbation of the test function $\boxed{p = \tau c\, w_{,x}}, \quad \tau = \Delta x/(2|c|)$

$$0 = \int_0^L (-w_{,x}c\phi - w f)\,dx + \sum_{n=1}^{N_{el}} \int_{x_{n-1}}^{x_n} \boxed{p(c\phi_{,x}} - f)\,dx, \quad \int_{x_{n-1}}^{x_n} \boxed{pc\phi_{,x}}\,dx \Leftrightarrow \boxed{\frac{|c|\Delta x}{2}\left(\frac{\phi_{n+1}-2\phi_n+\phi_{n-1}}{\Delta x^2}\right)}$$

- **Hyperbolic Systems of Conservation Laws:** Apply SUPG along characteristics

Conservation form: $\overbrace{\dot{U}(Y) + F_{i,i}(Y) + Z(Y) - \mathcal{F}}^{\mathcal{N}(Y)} = 0$

Quasi-linear form: $\underbrace{A_0\dot{Y} + A_i Y_{,i} + CY}_{\mathcal{L}Y} - \mathcal{F} \equiv Res\ Y = 0$

$$\mathcal{L}_{adv} = A_0\frac{\partial}{\partial t} + A_i\frac{\partial}{\partial X_i} \quad , \quad Res Y^h = \overbrace{(\mathcal{L}_{adv} + C)}^{\mathcal{L}} Y^h - \mathcal{F}$$

$$\mathcal{SUPG}(W^h, Y^h) = -\sum_{e=1}^{(n_{el})_n} \int_{Q_n^e} (\boxed{\mathcal{L}^*_{adv} W_h) \cdot \tau}\, Res(Y^h)\, dQ$$

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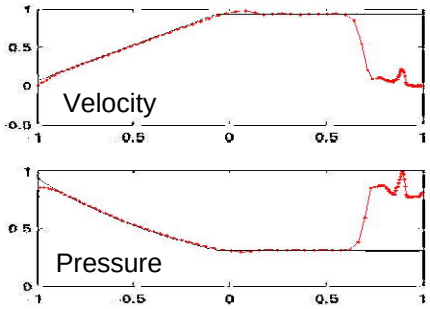
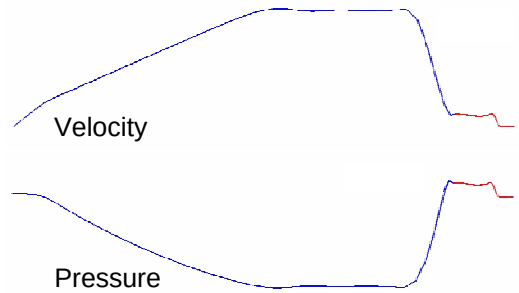
Results:
1D, 2D quadrilaterals, 2D triangles

- **Sod test: long-time behavior (T=0.9): interaction reflected shocks/contact**

Standard hydrocode:
CFL=0.9,
100 elements

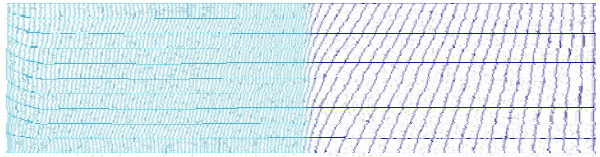


SUPG:
CFL=0.9
100 elements
2 corrector passes

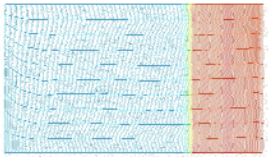


- **SUPG: Saltzmann test, quadrilaterals, 6 reflections!** Pressure: CFL=0.9, 2 corrector passes

before 1st reflection



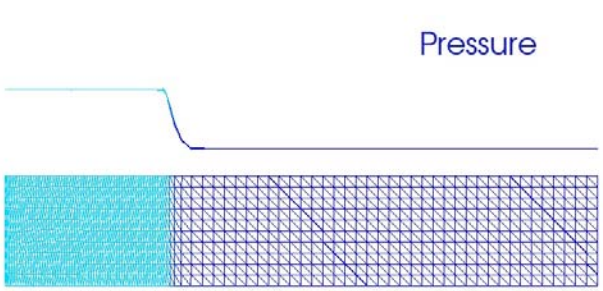
after 1st reflection

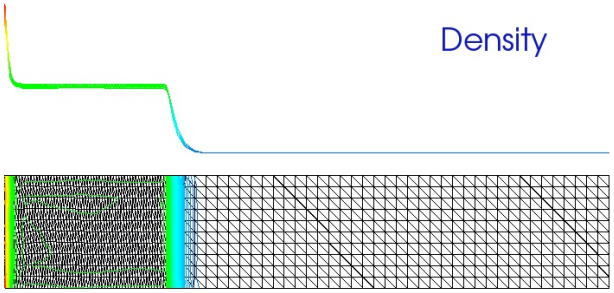


after 5th reflection

- **SUPG: Noh test, triangular mesh (aspect ratio 1:1, cartesian coordinates):**

CFL=0.75, 2 corrector passes, 2000 triangular elements





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Conclusion

- **Node-based uniform strain element technology (nodal based tets) for Lagrangian modeling is seen as an important option for rapid turn around of computations.**

- **SUPG approach for shock hydrodynamics appears to provide robust solutions on triangular meshes.**

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