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DS_N-METHOD WITH ARTIFICIAL DISSIPATION AND DMT-METHOD OF ITERATION ACCELERATION FOR THE NUMERICAL SOLUTION OF 2D HEAT TRANSFER EQUATIONS IN KINETIC APPROXIMATION

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Problem statement

$$\frac{1}{c}\frac{d\,J_g}{dt} + div\left(\vec{\Omega}\,J_g\right) + \alpha_g J_g = \frac{\alpha_{sg}}{4\pi}\int_{\vec{\Omega}} J_g\,d\vec{\Omega} + \frac{\alpha_{cg}}{4\pi}B_g(T),$$

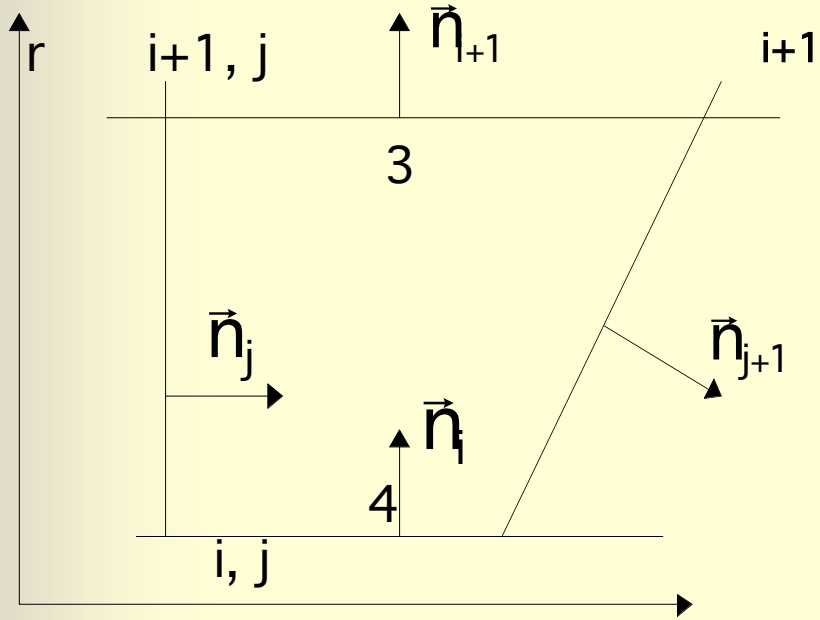
$$div\left(\vec{\Omega}J\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\xi J\right) + \frac{\partial}{\partial z}\left(\mu J\right) - \frac{1}{r}\frac{\partial}{\partial \phi}\left(\eta J\right),$$

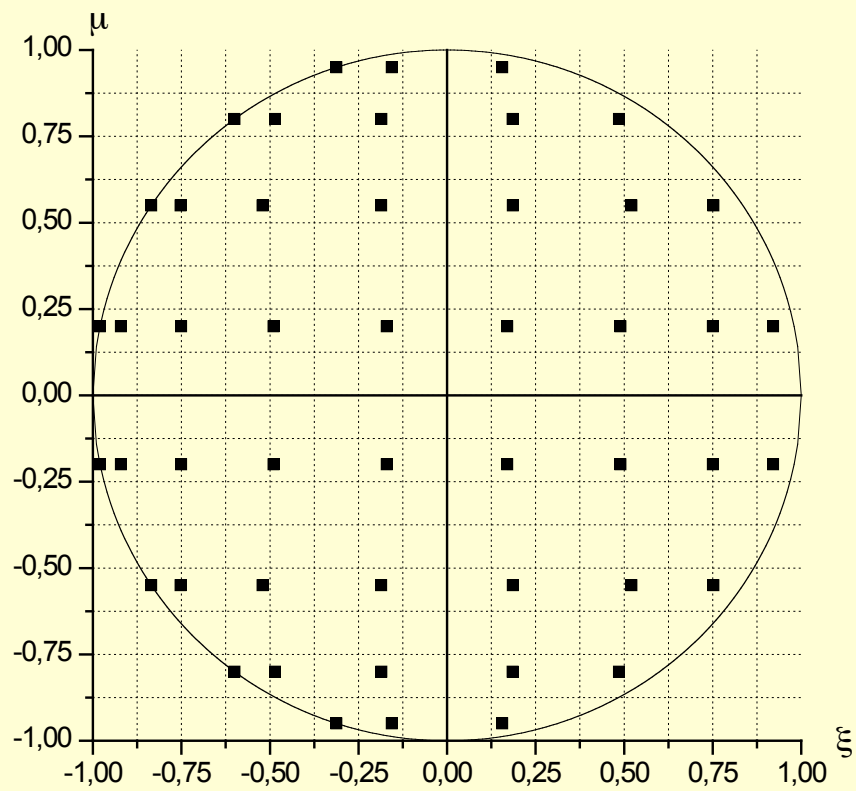
$$\rho\frac{\partial E}{\partial t} = \sum_{g=1}^{G}\alpha_{cg}\int_{\vec{\Omega}}\left(J_g - \frac{1}{4\pi}B_g\right)d\vec{\Omega} + \rho Q,$$

$$\mu = \cos\theta, \xi = \sqrt{1-\mu^2}\cos\phi, \eta = \sqrt{1-\mu^2}\sin\phi,$$

$$g = 1, 2, \ldots, G.$$

Approximation





Approximation

$$\frac{1}{c\Delta t^n}\left[(J_g)^{n+1}-(J_g)^n\right]+div_h(\vec{\Omega}J_g^{n+1})+\alpha_g J_g^{n+1}=Q_g^{n+1},$$

$$Q_g^{n+1}=\frac{1}{4\pi}\left(\alpha_{sg}U_g^{n+1}+\alpha_{cg}B_g^{n+1}\right),$$

$$J_g^{n+1}=(J_g)^{n+1}_{i+\frac{1}{2},j+\frac{1}{2},k-\frac{1}{2},m-\frac{1}{2}}$$

$$U_g=\sum_{m=1}^{M}\sum_{k=1}^{K_m}J_g\Delta\mu_{m-1}\Delta\phi_{k-1},$$

$$div_h(\vec{\Omega}J_g)\equiv\frac{1}{\Delta V_{ij}}\left[\Delta_i(\vec{\Omega}\vec{n}\Delta lrJ_g)+\Delta_j(\vec{\Omega}\vec{n}\Delta lrJ_g)\right]-\frac{\Delta S_{ij}}{\Delta V_{ij}}\frac{\Delta_k(\eta J_g)}{\Delta\phi_k},$$

$$(\vec{\Omega}\vec{n}\Delta l)_i=+\xi\Delta z_i-\mu\Delta r_i=+\xi(z_{i,j+1}-z_{ij})-\mu(r_{ij+1}-r_{ij}),$$

$$(\vec{\Omega}\vec{n}\Delta l)_j=-\xi\Delta z_j+\mu\Delta r_j=-\xi(z_{i+1,j}-z_{ij})+\mu(r_{i+1,j}-r_{ij}),$$

$$\tau=t^{n+1}-t^n,\ \ \Delta_i f=f_{i+1}-f_i,\ \ \Delta_j f=f_{j+1}-f_j,$$

Diamond Difference with the Artificial Dissipation (*the DDAD – scheme)*

$$\frac{1}{c\Delta t}\left[\left(J_g\right)^{n+1}-\left(J_g\right)^{n}\right]+div_h(\vec{\Omega}\Psi_g)+\alpha_g J_g=Q_g,$$

$$J_g=\Psi_g+\delta_g h div_h(\vec{\Omega}\Psi_g)-\theta_g h div_h\left(\vec{\Omega}\tilde{Q}_g\right) \quad J_g-\Psi_g=O\left(\Delta x^2\right)$$

$$\tilde{Q}_g=Q_g+\frac{1}{c\Delta t}\left(J_g\right)^{n}, \qquad \tilde{\alpha}_g=\frac{1}{c\Delta t}+\alpha_g,$$

$$\frac{1}{c\Delta t}\left[\left(J_g\right)^{n+1}-\left(J_g\right)^{n}\right]+div_h(\vec{\Omega}J_g)+\alpha_g J_g=Q_g+$$

$$+\delta_g h\, div_h(\vec{\Omega}J_g)-\theta_g h div_h\left(\vec{\Omega}\tilde{Q}_g\right)+o(h^2),$$

$$\tilde{\alpha}_g\Psi_g+\left(1+\delta_g h\tilde{\alpha}_g\right)div_h(\vec{\Omega}\Psi_g)=\tilde{\alpha}_g\left[\frac{\tilde{Q}_g}{\tilde{\alpha}_g}+\tilde{\alpha}_g\theta_g\Delta h div_h\left(\vec{\Omega}\frac{\tilde{Q}_g}{\tilde{\alpha}_g}\right)\right],$$

The DDAD - scheme

$$A: \quad \tilde{\alpha}_g J_g + div_h(\vec{\Omega} J_g) = \tilde{Q}_g, \qquad 0 < \tilde{\alpha}_g \Delta x < \infty,$$

$$B: \quad \alpha_g^* \Psi_g + div_h(\vec{\Omega} \Psi_g) = \alpha_g^* Q_g^*, \quad 0 < \alpha_g^* \Delta x \le 2,$$

$$Q_g^* = \frac{\tilde{Q}_g}{\tilde{\alpha}_g} + \tilde{\alpha}_g \theta_g h div_h \left(\vec{\Omega} \frac{\tilde{Q}_g}{\tilde{\alpha}_g} \right), \quad \alpha_g^* = \frac{\tilde{\alpha}_g}{1 + \delta_g h \tilde{\alpha}_g},$$

$$\delta_g = \frac{1}{2} \frac{1 + \exp\left(-\tilde{\alpha}_g \Delta x_s\right)}{1 - \exp\left(-\tilde{\alpha}_g \Delta x_s\right)} - \frac{1}{\tilde{\alpha}_g \Delta x_s}, \quad \theta_g = \frac{1}{\tilde{\alpha}_g} \delta_g, \qquad h = \Delta x_s$$

THE DIAGONAL MATRIX TECHNIQUE (DMT-METHOD) OF ITERATION ACCELERATION

$$\frac{1}{c\Delta t}\left[\left(J_g\right)^{n+1}-\left(J_g\right)^{n}\right]+div_h(\vec{\Omega}\Psi_g)+\alpha_g J_g=Q_g,$$

$$div_h\left(\vec{\Omega}\Psi\right)=\frac{1}{\Delta V_{ij}}a\Psi-\frac{1}{\Delta V_{ij}}\sum_{(\vec{\Omega}\vec{n})_l<0}b_l\Psi_l,$$

$$\left(\frac{1}{c\Delta t}+\alpha_g+d_g^{\nu+1/2}\right)U_g^{\nu+1/2}=\alpha_{sg}U_g^{\nu}+\alpha_{cg}B_g\left(T^{\nu}\right)+$$

$$+\frac{1}{\Delta V_{ij}}\left(\sum_{m=1}^{\grave{I}}\sum_{k=1}^{K_m}\left(\sum_{(\vec{\Omega}\vec{n})_l<0}b_l\Psi_l^{\nu+1/2}\right)_{g,k-1/2,m-1/2}\Delta\mu_{m-1}\Delta\phi_{k-1}\right)+\frac{\rho}{c\Delta t}\left(\frac{U_g}{\rho}\right)^{n},$$

$$\left(\frac{1}{c\Delta t}+\alpha_g+d_g^{\nu+1/2}\right)U_g^{\nu+1}=\alpha_{sg}U_g^{\nu}+\alpha_{cg}B_g\left(T^{\nu+1}\right)+$$

$$+\frac{1}{\Delta V_{ij}}\left(\sum_{m=1}^{\grave{I}}\sum_{k=1}^{K_m}\left(\sum_{(\vec{\Omega}\vec{n})_l<0}b_l\Psi_l^{\nu+1/2}\right)_{g,k-1/2,m-1/2}\Delta\mu_{m-1}\Delta\phi_{k-1}\right)+\frac{\rho}{c\Delta t}\left(\frac{U_g}{\rho}\right)^{n},$$

$$d_g^{\nu+1/2}=\frac{\frac{1}{\Delta V_{ij}}\sum_{m=1}^{\grave{I}}\sum_{k=1}^{K_m}\left(a\Psi_g^{\nu+1/2}\right)_{k-1/2,m-1/2}\Delta\mu_{m-1}\Delta\phi_{k-1}}{U_g^{\nu+1/2}}$$

DMT-METHOD OF ITERATION ACCELERATION

$$U_g^{\nu+1} = U_g^{\nu+1/2} + b_g(B_g^{\nu+1} - B_g^{\nu}), \quad b_g = \frac{\alpha_{cg}}{\dfrac{1}{c\Delta t} + \alpha_{cg} + d_g^{\nu+1/2}},$$

$$T^{k+1} = T^k + \Bigg\{ \rho(E^n - E^k) + \Delta t^n \rho Q +$$

$$+ \Delta t^n \sum_{g=1}^{G} \left[\alpha_{cg} \left(U_g^{\nu+1/2} + b_g(B_g(T^k) - B_g^{\nu}) \right) - \alpha_{cg} B_g(T^k) \right] \Bigg\} \times$$

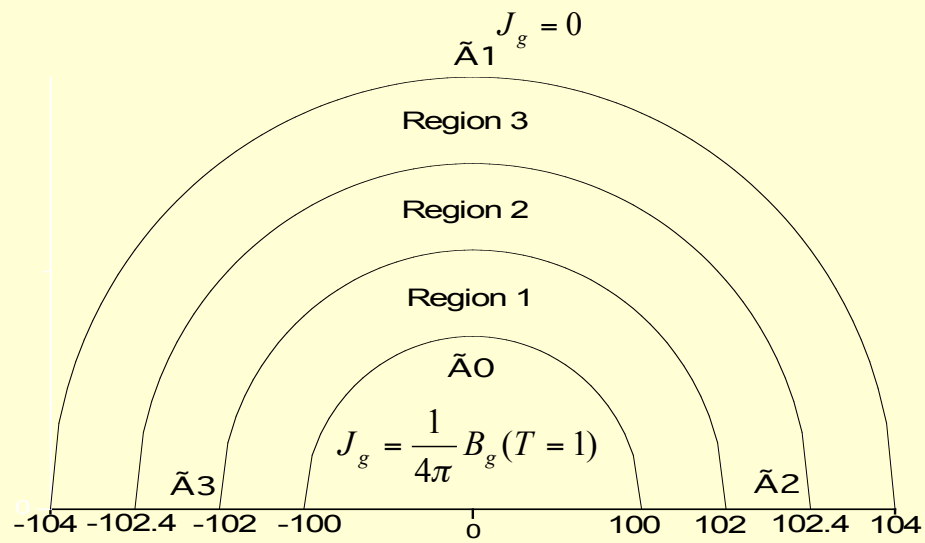
$$\times \left\{ \rho E_T^k + \Delta t \sum_{g=1}^{G} \left[\alpha_{cg}(1 - b_g) \right] B_{gT}^k \right\}^{-1}, \quad \text{ãäå}\ E_T = \left(\frac{\partial E}{\partial T} \right)^k, B_{gT} = \left(\frac{\partial B_g}{\partial T} \right)^k.$$

The Fleck problem

$$\alpha_c = \frac{\chi(1-e^{-\varepsilon/T})}{\varepsilon^3},$$

χ = 27 in regions 1 and 3,
χ = 10000 in region 2.

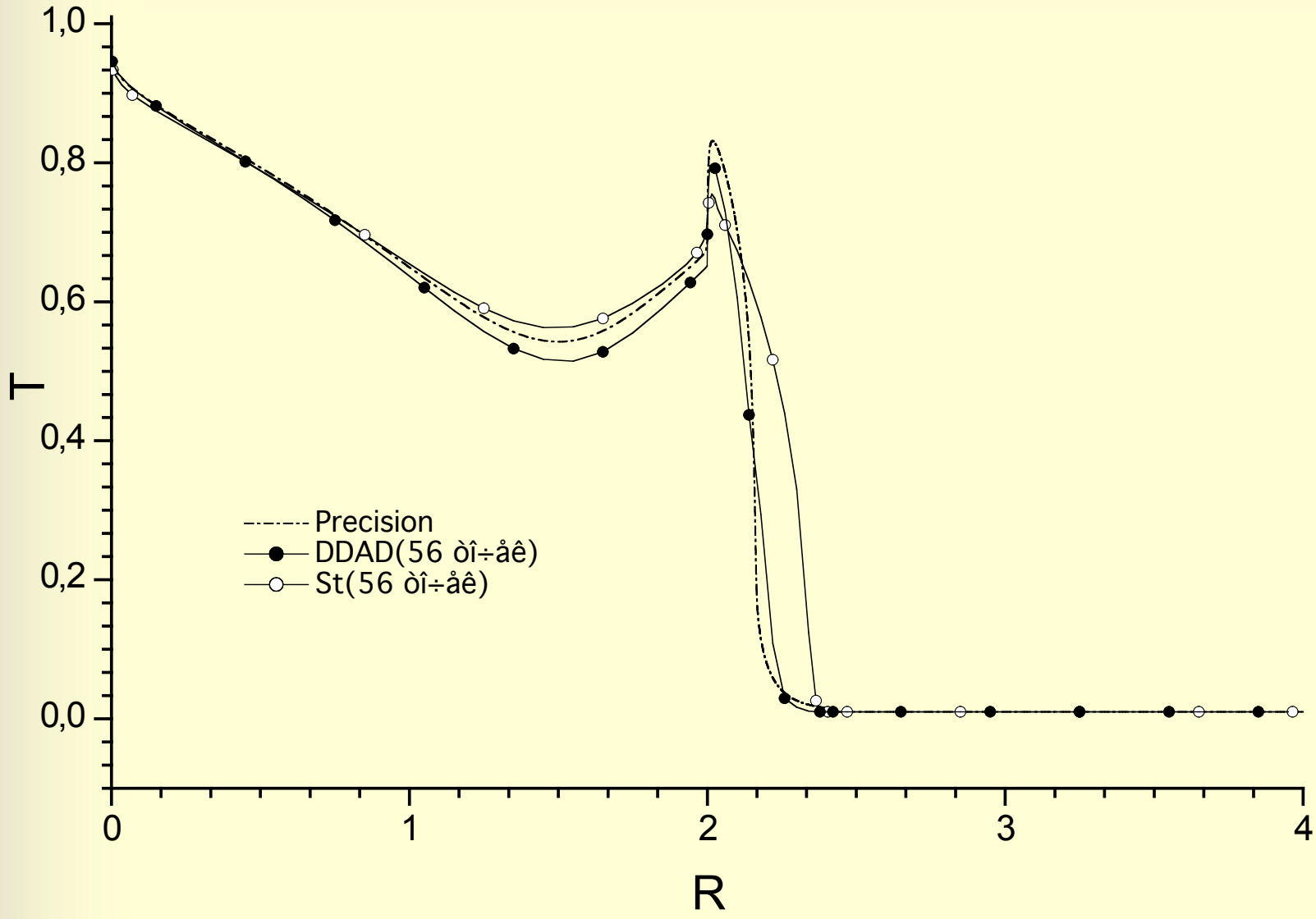
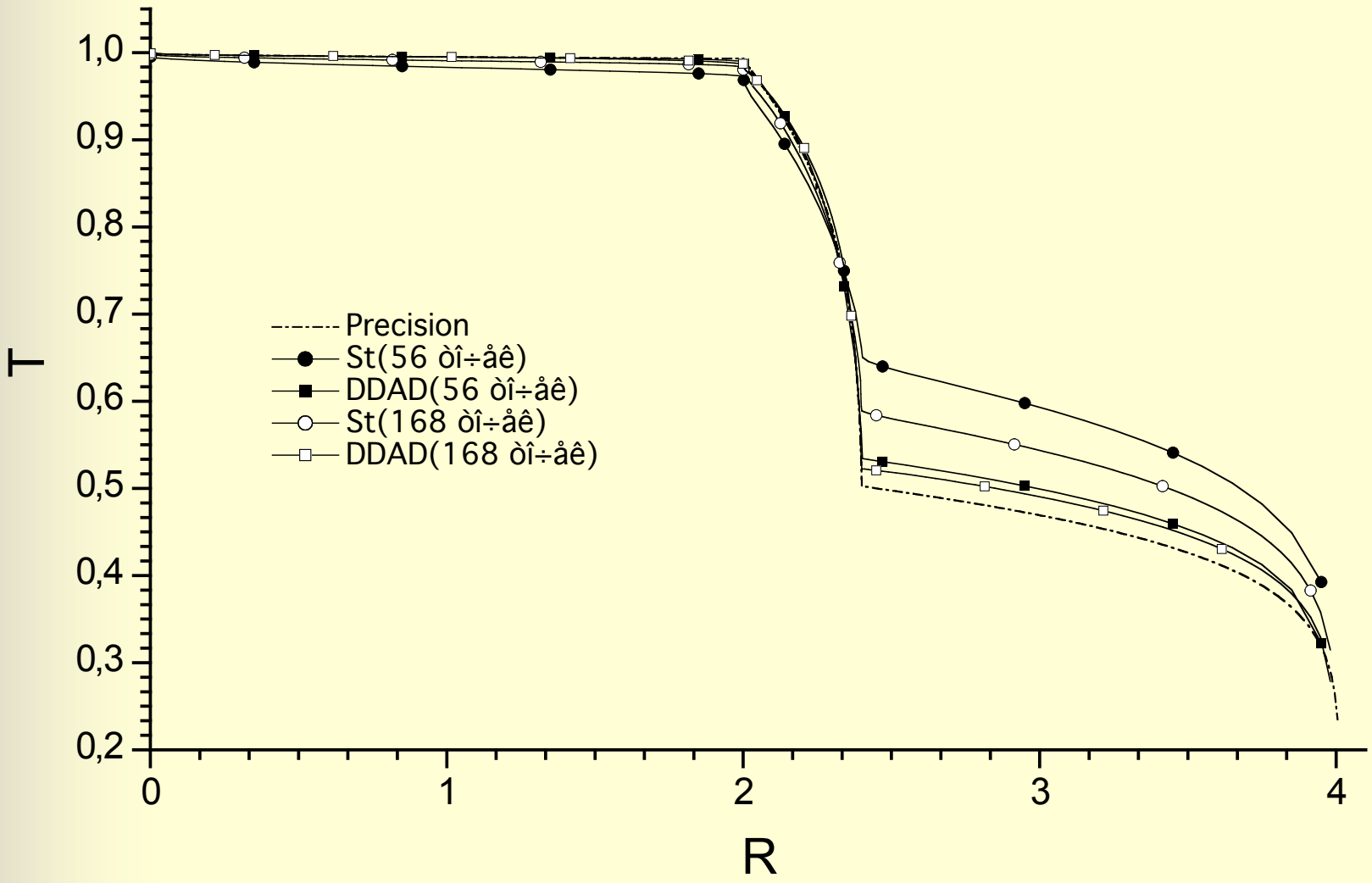
$E_{1,2,3}$ = 0.81T



$$J_g(t,0,z,\mu,\psi) = J_g(t,0,z,\mu,\pi-\psi), \quad \psi < \frac{\pi}{2}$$

ε_g =15.0, 12.0, 10.0, 8.0, 7.0, 6.0, 5.5, 5.0, 4.5, 4.0, 3.5, 3.0, 2.6, 2.2, 1.8, 1.4, 1.0, 0.7, 0.5, 0.4, 0.3, 0.2, 0.15, 0.1, 0.08, 0.06, 0.04, 0.02, 0.0 (28 groups).

The Fleck problem



The Fleck problem