Reformulation of the Analytic Solution of the Elastic-Plastic Dynamic Sphere Test Problem

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ABSTRACT

An analytic solution is reformulated for the dynamic response of a spherical shell subjected to symmetric, time variant loading conditions. The constitutive relation incorporates a linear-mechanical stress and eigenstresses, which account for history-dependent effects. The present implementation considers an isotropic hardening, elastic-plastic material model. The initial shell configuration is of finite radius, and the solution is limited to the small-strain regime. Time dependent Dirichlet, Neumann or mixed boundary conditions are prescribed on the shell inner and outer surface, together with initial displacement and velocity fields through the shell. The analytic solution uses a Lagrangian approach, i.e., displacements are defined relative to an initial configuration so that mass conservation is inherently satisfied. The temperature is uniform in space and constant in time so that the energy equation can be neglected. Conservation of momentum provides the single governing equation of motion.

The present reformulated solution and corresponding numerical implementation extends methods used to solve a similar problem that was limited to linear-elastic materials. [1, 2] The solution for history dependent materials, e.g., elastic-plastic solids, requires additional mathematical treatments to the elastic solution. In particular, the inelastic material solution evolves eigenstress tensor components at discrete time and space locations. The original analytic solution, however, includes spatial gradients and time derivatives of the eigenstress tensor, which are problematic to evaluate for discrete data. To obviate eigenstress spatial gradients, integration by parts and a recursion relationship for Bessel function derivatives are applied. Integration by parts is used again to eliminate the second time derivative of the eignestress tensor. Integration by parts is used a third time to minimize, but not eliminate, the occurrence of eigenstress first time derivatives. The solution forcing function requires evaluation of multiple integrals involving these discrete data and, thus, quadrature methods are dependent upon the solution time and space grid.

The solution method is implemented in a computer code where the algorithm includes a truncated series of weighted Bessel functions to numerically approximate the infinite-series solution. When the quantitative impact of truncating the infinite series is small compared to the solution variables and small compared to the difference between analytic and simulation results, the solution is considered to be ‘exact’. Such benchmark solutions represent valuable tools for quantifying the accuracy of and, thus, verifying computational plasticity simulation codes.

REFERENCES

[1] T.O. Williams, S. Li, J.S. Brock and J.R. Kamm, “Spherical Shell Analysis for Material Modeling,” Los Alamos National Laboratory, LA-UR-05-8038, (2005).

[2] J.R. Kamm, T.O. Williams, J.S. Brock and S. Li, “Application of Gegenbauer Polynomial Expansions to Mitigate Gibbs Phenomenon in Fourier-Bessel Series Solutions of a Dynamic Sphere Problem,” *Communications in Numerical Methods in Engineering*, 2009. (Los Alamos National Laboratory, LA-UR-08-3572, 2008.)

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